Forward-Link Capacity of a DS/CDMA System with Mixed Multirate Sources

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Abstract—Some studies have been done on capacity of a code division multiple access (CDMA) system with mixed multirate sources. However, a vast majority of these studies have concentrated on the reverse link. This trend comes from the fact that the capacity of a CDMA system is reverse-link limited. However, forward link can be a limiting link because emerging data services are likely to require higher data rates in the forward link than in the reverse link. In this paper, we analyze and simulate the forward-link capacity of a CDMA system with mixed multirate sources in a multipath fading channel. The outage probability of the forward link is derived for a CDMA system with mixed multirate sources. By introducing a forward-link power factor, the forward-link Erlang capacity is obtained in a closed form. The forward-link capacity is analyzed in terms of the number of multipaths, the number of rake fingers in a mobile station, closed-loop power control, and impact of soft handoff. The results in this paper can be applied to overall system design of a CDMA system with multimedia services in future mobile communication systems.

Index Terms—Direct sequence code division multiple access (DS/CDMA), forward-link capacity, multirate sources.

I. INTRODUCTION

Demand for wireless communications and internet applications is continuously growing. These days, the majority of internet services are provided via wireline. It is expected that Internet service via wireless medium will dramatically increase in the near future. Existing code division multiple access (CDMA) cellular systems are designed to support for voice and low-speed data services, but not for high-rate data service. A recently evolved and implemented CDMA standard, IS-95B [1], can support up to 115.2-kb/s data service by using multiple code. In the IS-95B system, up to eight codes can be assigned to a single data service user for higher rate transmission, and a fundamental code channel must remain assigned to the data service user for entire call duration even though the data service user has nothing to transmit/receive. However, the IS-95B system is not sufficient to cover emerging data services in terms of spectrum and coverage efficiency. Therefore, third-generation wireless systems such as international mobile telecommunications—2000, having the ability to support high-rate data services with high spectrum efficiency and high coverage efficiency, are considered [2].

The capacity of a system can be defined in several ways. Its typical meaning is the maximal number of simultaneous users, and this definition is used to design a call-admission control for the given set of capacity lines. The other definition is peak load that can be supported by the system while maintaining desired service quality. The latter definition can be used to find Erlang capacity that is used in cell planning and in evaluating capacity expense.

In previous research, studies have been done on the capacity of a CDMA system with mixed multirate sources [3]–[10]. The Erlang capacity of the reverse link with mixed multirate sources was investigated in [3]–[5]. The maximal number of simultaneous users of the reverse link with multiclass traffic was studied in [6]–[8]. However, these studies have concentrated on the reverse link. This trend comes from the fact that the capacity of a CDMA system is reverse-link limited. However, forward link can be a limiting link because emerging data services are likely to require higher data rates in the forward link than in the reverse link. The forward-link capacity of a CDMA system with mixed multirate sources was studied in [9] and [10]. However, these studies are based on the computer simulation rather than mathematical analysis. So far, there have been few studies to derive the forward-link capacity of a CDMA system with mixed multirate sources in a closed form.

In this paper, the forward-link capacity of the CDMA system with mixed multirate sources is analyzed and simulated in a multipath fading channel. By introducing a forward-link power factor, the forward-link Erlang capacity and outage probability is obtained in a closed form. The forward-link capacity is analyzed in terms of the number of multipaths, the number of rake fingers in a mobile station, closed-loop power control, and impact of soft handoff.

The rest of this paper is organized as follows. Data traffic and interference models are described in Section II. Forward-link outage probability and Erlang capacity are derived in Section III. Numerical results are presented in Section IV, and conclusions are drawn in Section V.

II. SYSTEM MODEL

A. Data Traffic Model

In the source level, data traffic is commonly modeled as an ON–OFF source [11]. This model basically consists of two major sets of parameters: 1) distributions of ON–OFF periods and 2) distribution of packet arrivals during an ON period. In this paper, for simplicity of analysis, we assume continuous transmission during a burst or ON-period. Recent studies on intranet and Internet traffic indicate that although the ON–OFF source model is appropriate for data packet, the distributions of the ON–OFF packets
sources could have infinite variance, as opposed to the finite variance assumption in the traditional traffic model [12].

A heavy-tailed distribution with infinite variance called Pareto distribution is known to match well with the actual data traffic measurements in the application, the source, and the aggregate levels. Recent traffic measurements show that the holding time of a data service user (i.e., data call duration) can also be modeled by a heavy-tail distribution (such as Pareto distribution), and the arrival of data call remains Poisson [i.e., the interarrival time (idle time) can be modeled by an exponential distribution]. The probability density function (pdf) of a Pareto distribution is given by

$$ f(t) = \frac{\alpha \beta^\alpha}{(t + \beta)^{\alpha+1}} $$

where expectation of the Pareto distribution is given by $E[t] = \beta/(\alpha - 1)$.

The Pareto distribution has two parameters $\alpha$ and $\beta$. The $\alpha$ represents “heaviness” of the tail of distribution. When the $\alpha$ is closer to one, the distribution tail becomes heavier and the traffic becomes more bursty. Once a suitable value for the $\alpha$ is selected according to the data characteristic, the $\beta$ can then be set based on the mean of the distribution. If the $\alpha$ is between one and two, the variance of distribution becomes infinity. The variance becomes finite if the $\alpha$ is two or above.

The data traffic can be characterized by an activity factor that is defined as a duty cycle (percentage of time in which the user receives/transmits information during his call duration). From the data traffic pattern, as shown in Fig. 1, the activity factor $\rho_d$ can be obtained as

$$ \rho_d = \frac{T_{\text{active holding time}}}{T_{\text{holding time}}} = \frac{\sum N T_{\text{on}}}{\sum N T_{\text{on}} + \sum N T_{\text{off}}} \approx \frac{E[T_{\text{on}}]}{E[T_{\text{off}}] + E[T_{\text{on}}]} $$

where

- $N$ average number of sessions per a data call;
- $T_{\text{on}}$ duration of transmitting data;
- $T_{\text{off}}$ time when there is no data transmission;
- $E[.]$ expectation value.

$T_{\text{on}}$ depends on file size to be transmitted during the session and the transmission rate, while $T_{\text{off}}$ is related to the human–computer interaction and server response time.

B. Interference Model

Path loss between a mobile station (MS) and the $m$th base station (BS) is given by

$$ L_m = D_m^l \cdot 10^{\xi_m/10} $$

where

- $D_m$ distance between the $m$th BS and the MS;
- $l$ path-loss exponent (typically three to four);
- $\xi_m$ Gaussian distributed random variable with zero mean and standard deviation (std) $\sigma$, representing shadow fading;
- $\chi_m$ lognormally distributed random variable.

Typically, the $\sigma$ is between 6–10 dB for the signals from adjacent BSs and 2–2.5 dB for the signals from home BS when closed-loop fast power control is employed [13]–[15].

In our system model, the following assumptions are made:

1) $K$ multiple cells are uniformly loaded;
2) cell pattern is hexagonal;
3) data service users are classified into $M$ classes in each cell;
4) each MS receives $J$ multipath rays from each BS;
5) full orthogonality in forward-link code channels is not guaranteed because of multipath;
6) the receiver of MS has $J$ rake receivers.

Then, the intracell interference power on the $j$th finger of the MS is given by

$$ I_{sc} = \sum_{m=1}^{J} \alpha^{(m)} \cdot (1 - \psi \cdot \phi_k) \cdot P_0 \cdot L_0 $$

where

- $\alpha^{(j)}$ power portion of the $j$th multipath so that $\sum_{j=1}^{J} \alpha^{(j)} = 1$,
- $\psi$ fraction of BS power assigned to traffic channels;
relative power for the \( i \)th user;
\( P_0 \) total transmitted signal power from home BS.

The intercell interference power from adjacent BSs to the MS is given by

\[
I_{IC} = \sum_{k=1}^{K} \sum_{n=1}^{J} \alpha^{(n)} \cdot P_k \cdot L_k \quad (5a)
\]

\[
\approx \sum_{k=1}^{K} P_k \cdot L_k \quad (5b)
\]

where \( P_k \) is the total transmitted signal power from the \( k \)th BS. Then, the received \( E_k/N_t \) at the MS is given by (6a) and (6b), shown at the bottom of the page, where

\( W \) spreading bandwidth;

\( R \) data rate;

\( I_{IC} = \sum_{k=1}^{K} \sum_{n=1}^{J} \alpha^{(n)} \cdot P_k \cdot L_k; \)

\( I_{SC} = \sum_{n=1}^{J} \alpha^{(n)} \cdot (1 - \psi \phi_i) \cdot P_0 \cdot L_0. \)

The transmitted signal power of a BS is actually a function of the number of ongoing calls served by the BS and thus, generally, can be modeled as a random variable. However, in this paper, we assumed that all BSs are accommodating maximal ongoing calls and transmitting signals at full power level. This corresponds to the worst case condition.

III. FORWARD LINK CAPACITY

In this section, we derive forward-link outage probability and evaluate the capacity. The background noise is negligible compared to the total signal power. Hence, from (6a) and (6b), the received \( E_k/N_t \) at the MS is given by

\[
\gamma \approx \frac{W \cdot \psi}{R} \sum_{j=1}^{J} \frac{\phi_i \cdot \alpha^{(j)}}{S_0} + (1 - \alpha^{(j)}) \cdot (1 - \psi \phi_i) \quad (7)
\]

where \( S_0 = P_0 \cdot L_0 \) denotes total received power at the MS from home BS.

As shown in Fig. 2, in the worst case, i.e., the MS is at cell boundary, the ratio of intercell interference power to received power at the MS from the home BS is given by

\[
\frac{I_{IC}}{S_0} = \frac{\sum_{k=1}^{K} P_k \cdot L_k}{\sum_{k=1}^{K} \frac{P_k \cdot L_k}{P_0 \cdot L_0}} \quad (8a)
\]

\[
= \sum_{k=1}^{K} \left( \frac{D_k}{D_0} \right)^{-1} \cdot \frac{\chi_k}{\chi_0} \quad (8b)
\]

where \( D_k \) is the distance from the \( k \)th BS to the MS and \( \chi_k \) is a lognormal random variable representing shadow fading from the \( k \)th BS. Since the sum of lognormal random variables can be approximated as a lognormal random variable [12], the \( I_{IC}/S_0 \) can be approximated as a lognormal random variable with a mean dB value \( m_y \) and a standard deviation of dB value \( \sigma_y \) (refer to Appendix A).

Since \( \psi \phi_i \ll 1 \), the received \( E_k/N_t \) of (7) can be approximated as

\[
\gamma \approx \frac{W \cdot \psi \cdot J^{(0)}}{R} \sum_{j=1}^{J} \frac{\phi_i \cdot \alpha^{(j)}}{S_0} + (1 - \alpha^{(j)}) \cdot (1 - \psi \phi_i) \quad (9)
\]

From (9), we can find that the received \( E_k/N_t \) at the MS is different according to data rate \( R \) and relative power for the \( i \)th user \( \phi_i \). If the \( d_i, \ i = 1, \ldots, M \), denotes data service class,
then the received $E_b/N_0$ of voice user $d_1$-class data service user $\ldots$, and $d_M$-class data service user are as follows:

$$
\gamma_0 \approx \frac{W \psi}{R_w} \sum_{j=1}^{J} \frac{\phi_i^{(v)} \cdot \alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{10}
$$

$$
\gamma_{d_k} \approx \frac{W \psi}{R_{d_k}} \sum_{j=1}^{J} \frac{\phi_i^{(d_k)} \cdot \alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{11}
$$

$$
\gamma_{d_M} \approx \frac{W \psi}{R_{d_M}} \sum_{j=1}^{J} \frac{\phi_i^{(d_M)} \cdot \alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{12}
$$

### A. Outage Probability

We assume that there are $K_v$ voice users, $K_{d_k}$, $d_k$-class data service users $\ldots$, and $K_{d_M}$, $d_M$-class data service users in a cell. We also assume that $K_v, K_{d_k}, \ldots, K_{d_M}$, are Poisson random variables with call arrival rates $\lambda_w, \lambda_{d_k}, \ldots, \lambda_{d_M}$, respectively, and average call durations $\mu_v, \mu_{d_k}, \ldots, \mu_{d_M}$, respectively.

In (11) and (12), the relative power for the $i$th user being served is given by

$$
\phi_i^{(v)} = \frac{\gamma_0 \cdot R_w}{W \psi} \cdot \sum_{j=1}^{J} \frac{\alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{13}
$$

$$
\phi_i^{(d_k)} = \frac{\gamma_{d_k} \cdot R_{d_k}}{W \psi} \cdot \sum_{j=1}^{J} \frac{\alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{14}
$$

$$
\phi_i^{(d_M)} = \frac{\gamma_{d_M} \cdot R_{d_M}}{W \psi} \cdot \sum_{j=1}^{J} \frac{\alpha_j}{\log_2 e} + \left(1 - \alpha_j \right) \tag{15}
$$

For the worst case that all the MSs are at the cell boundary, we introduce an average forward-link power factor $\eta$ in order to take into account the fact that most of the MSs are not located at the cell boundary. If all the MSs are uniformly distributed at each cell, and fourth power law and perfect power control are applied, the $\eta$ is about 0.4 (refer to Appendix B). The average forward-link traffic channel power is about $\eta$ times the traffic channel power needed to serve the MS at the cell boundary. Then, outage probability is given by (16a)–(16c), shown at the bottom of the page, where $\rho_i$ denotes activity factor of service class ($\cdot$).

### B. No Multipath (Only a Direct Path) Case

We assume that there is no multipath component, that is, only a direct path exists. Since $J'(0) = 1$ and $\alpha'(0) = 1$ for this case, $\phi_i^{(v)}, \phi_i^{(d_k)}, \ldots, \phi_i^{(d_M)}$ in (13)–(15) are modified, respectively, as follows:

$$
\phi_i^{(v)} = \frac{\gamma_0 \cdot R_w}{W \psi} \cdot \frac{S_0}{L \psi} \tag{17}
$$

$$
\phi_i^{(d_k)} = \frac{\gamma_{d_k} \cdot R_{d_k}}{W \psi} \cdot \frac{S_0}{L \psi} \tag{18}
$$

$$
\phi_i^{(d_M)} = \frac{\gamma_{d_M} \cdot R_{d_M}}{W \psi} \cdot \frac{S_0}{L \psi} \tag{19}
$$

where $\gamma_i = (L_i x S_0)$ is lognormally distributed random variable with mean dB value $m_\gamma$ and standard deviation of dB value $\sigma_\gamma$.

From (17)–(19), the outage probability is derived as

$$
P_{out} = \Pr \left[ \eta \left( \sum_{i=1}^{K_v} \rho_i^{(v)} y_i + \sum_{i=1}^{K_{d_k}} \rho_i^{(d_k)} y_i + \ldots + \sum_{i=1}^{K_{d_M}} \rho_i^{(d_M)} y_i \right) > \frac{\psi}{\eta} \right] \tag{20a}
$$

$$= \Pr \left[ Z = Z_v + Z_{d_k} + \ldots + Z_{d_M} > \frac{\psi}{\eta} \right] \tag{20b}
$$

where $Z_v = \gamma_0 R_w / \sum_{i=1}^{K_v} \rho_i^{(v)} y_i$, $Z_{d_k} = \gamma_{d_k} R_{d_k} / \sum_{i=1}^{K_{d_k}} \rho_i^{(d_k)} y_i$, $\ldots$, and $Z_{d_M} = \gamma_{d_M} R_{d_M} / \sum_{i=1}^{K_{d_M}} \rho_i^{(d_M)} y_i$ respectively.

Since $y_i$ ($i = 1, 2, \ldots, K_v$, or $K_{d_k}$ or $K_{d_M}$) are independent identically distributed lognormal random variables, we can approximate $Z_v, Z_{d_k}, \ldots, Z_{d_M}$ as lognormally distributed random variables conditioned on the number of active voice and data service users, respectively [16]. The random variable $Z = Z_v + Z_{d_k} + \ldots + Z_{d_M}$ can be approximated as a lognormal random variable. It is well known that the lognormal approximation is more accurate than Gaussian approximation when the number of users is small. The mean and variance of $Z$ can be obtained from [4]

$$
E[Z] = \frac{\gamma_0 R_w}{W \psi} \cdot v \cdot f(m_\gamma, \sigma_\gamma)
$$

$$+ \frac{\gamma_{d_k} R_{d_k}}{W \psi} \cdot d_k \cdot f(m_\gamma, \sigma_\gamma)
$$

$$+ \ldots + \frac{\gamma_{d_M} R_{d_M}}{W \psi} \cdot d_M \cdot f(m_\gamma, \sigma_\gamma) \tag{21}
$$

$$P_{out} = \Pr \left[ \eta \left( \sum_{i=1}^{K_v} \rho_i^{(v)} y_i + \sum_{i=1}^{K_{d_k}} \rho_i^{(d_k)} y_i + \ldots + \sum_{i=1}^{K_{d_M}} \rho_i^{(d_M)} y_i \right) > 1 \right] \tag{16a}
$$

$$= \Pr \left[ \eta \left( \sum_{i=1}^{K_v} \rho_i^{(v)} y_i + \sum_{i=1}^{K_{d_k}} \rho_i^{(d_k)} y_i + \ldots + \sum_{i=1}^{K_{d_M}} \rho_i^{(d_M)} y_i \right) > \frac{1}{\eta} \right] \tag{16b}
$$

$$= \Pr \left[ Z > \frac{1}{\eta} \right] \tag{16c}
$$
\[
\text{Var}(Z) = \left( \frac{\gamma_v \cdot R_v}{W} \right)^2 V \cdot h(m_y, \sigma_y) + \left( \frac{\gamma_{d_1} \cdot R_{d_1}}{W} \right)^2 d_1 \cdot h(m_y, \sigma_y) + \cdots + \left( \frac{\gamma_{d_M} \cdot R_{d_M}}{W} \right)^2 d_M \cdot h(m_y, \sigma_y)
\]

where \( f(a, b) = \exp \{ \beta a + \beta b^2 / 2 \}, \beta = \ln(10) / 10 \), and \( h(a, b) = \exp \{ 2 \beta a + \beta b^2 \} \cdot (\exp \{ \beta b^2 \} - 1) \).

Then, the outage probability in (20b) can be derived as

\[
P_{\text{out}} = \Pr \left[ Z = \ln(Z) > \ln \frac{\psi}{\eta} \right] K_v K_{d_1} \cdots K_{d_M}
\]

\[
e^{\left( \frac{\psi \Lambda_{v}(1+g) + \Lambda_{d_1}(1+g) + \cdots + \Lambda_{d_M}(1+g)}{\mu_{d_1}} \right) - \ln \left( \frac{\psi}{\eta} \right)}
\]

\[
\times \sum_{i=1}^{\infty} \left( \Lambda_{v}(1+g) / \mu_{d_1} \right)^i i!
\]

\[
\times \sum_{d_1=0}^{\infty} \left( \Lambda_{d_1}(1+g) / \mu_{d_1} \right)^d d_1!
\]

\[
\sum_{d_M=0}^{\infty} \frac{Q \left( \ln(\psi/\eta) - E[Z] \right)}{\text{Var}(Z)} \left( \Lambda_{d_M}(1+g) / \mu_{d_M} \right)^d_M d_M!
\]

\]

where \( Z \) denotes \( \ln(Z) \) and is a Gaussian random variable with mean \( E[Z] \) and variance \( \text{Var}(Z) \). Equation (23b) follows from the fact that the number of active voice users \( K_v \) and the data service users \( K_{d_1}, \ldots, K_{d_M} \) are Poisson distributions with mean \( \rho_v \Lambda_{v}(1+g) / \mu_{d_1} \) and \( \rho_d \Lambda_{d_i}(1+g) / \mu_{d_i} \), respectively. In (23), for each service class user, \( \lambda_i \) is a call arrival rate, \( \mu_i \) is an average call duration, and \( g \) is a fraction of all the MSs that are in soft handoff. The MSs in soft handoff contribute approximately twice as much interference as MSs that are not in soft handoff. We assume that a fraction \( g < 1 \) of all the MSs are in soft handoff, and both BSs involved in soft handoff allocate essentially the same fraction to that MS. Then, the effective arrival rate per BS rises from \( \lambda \) to \( \lambda(1+g) \).

The relationships for evaluating \( E[Z] \) and \( \text{Var}(Z) \) are given by [12]

\[
E[Z] = \exp \left\{ E[Z] + \frac{1}{2} \text{Var}(Z) \right\}
\]

\[
\text{Var}(Z) = \ln \left( \text{Var}(Z) / E[Z] + 1 \right).
\]

C. Multipath Case

The outage probability cannot be obtained in a closed form for the case with multipath components. So, we employ Chernoff bound on the outage probability of (16). Then, the outage probability is bounded by

\[
P_{\text{out}} < E \left[ \exp \left\{ \sum_{i=1}^{K_v} s \cdot \rho_i^{(v)} \cdot \gamma_v R_v / W \right\} / Y \right]
\]

\[
+ \sum_{i=1}^{K_{d_1}} s \cdot \rho_i^{(d_1)} \cdot \gamma_{d_1} R_{d_1} / W \right\} / Y \right]
\]

\[
+ \cdots + \sum_{i=1}^{K_{d_M}} s \cdot \rho_i^{(d_M)} \cdot \gamma_{d_M} R_{d_M} / W \right\} / Y - s \psi / \eta \right\}
\]

where \( Y \) is defined by

\[
Y = \sum_{j=1}^{J(1)} \frac{\lambda(1)}{\eta_j} + (1 - \lambda(1))
\]

\[
= \sum_{j=1}^{J(1)} \frac{1}{\alpha_j} \left( \frac{\lambda(1)}{\eta_j} + (1 - \lambda(1)) \right)
\]

\[
= \sum_{j=1}^{J(1)} \frac{1}{\eta_j}.
\]

Since \( I \) is a lognormal random variable with mean dB value \( m_y \) and standard deviation of dB value \( \sigma_y \),

\[
(1 / \eta_j) (I_{\text{cc}} / S_0) \]

is also a lognormal random variable with mean dB value \( m_y - 10 \log(\gamma) \) and standard deviation of dB value \( \sigma_y \). \( (1 / \eta_j) \) is also a lognormal random variable with mean \( (1 / \eta_j) E[I_{\text{cc}} / S_0] + (1 / \eta_j) (I_{\text{cc}} / S_0) \) and standard deviation \( (1 / \eta_j) \text{Var}[I_{\text{cc}} / S_0] \).

Since \( K_v, K_{d_1}, \ldots, K_{d_M}, \rho_v^{(v)}, \rho_i^{(d_1)}, \ldots, \rho_i^{(d_M)} \), \( Y \) are mutually independent, the Chernoff bound on outage probability is given by

\[
P_{\text{out}} < \min_{\alpha > 0} E \left[ \exp \left\{ \sum_{i=1}^{K_v} s \cdot \rho_i^{(v)} \cdot \gamma_v R_v / W \right\} \right]
\]

\[
\cdot E \left[ \exp \left\{ \sum_{i=1}^{K_{d_1}} s \cdot \rho_i^{(d_1)} \cdot \gamma_{d_1} R_{d_1} / W \right\} \right]
\]

\[
\cdots \cdot E \left[ \exp \left\{ \sum_{i=1}^{K_{d_M}} s \cdot \rho_i^{(d_M)} \cdot \gamma_{d_M} R_{d_M} / W \right\} \right] \cdot e^{-s \psi / \eta}
\]

The Erlang capacity is the set of values \( \{ \lambda_v / \mu_v, \ldots, \lambda_{d_M} / \mu_{d_M} \} \) that keep the outage probability at a target level, typically 0.01.
Fig. 3. Forward-link Erlang capacity of cdma2000 (chip rate $= 1.2288$ Mc/s) with no multipath component.

\[ \rho_{dM} \lambda_d (1 + g)/\mu_{dM}, \text{respectively, the Chernoff bound on the} \]

outage probability is derived as (refer to Appendix C)

\[ P_{\text{out}} < \min_{s > 0} e^{\rho_{dM} \lambda_d (1 + g)/\mu_{dM} s} E_Y \left[ e^{\frac{\sigma_{dM}^2 R_{dM}}{R_{dM}}} - 1 \right] \]

\[ \times e^{\rho_{dM} \lambda_d (1 + g)/\mu_{dM}} E_Y \left[ e^{\frac{\sigma_{dM}^2 R_{dM}}{R_{dM}}} - 1 \right] \]

\[ e^{\rho_{dM} \lambda_d (1 + g)/\mu_{dM}} E_Y \left[ e^{\frac{\sigma_{dM}^2 R_{dM}}{R_{dM}}} - 1 \right] \cdot e^{-\psi} \]  

(29)

where $E[Y] \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot $ denotes expectation on random variable $Y$.

The Chernoff bound on the outage probability in (29) can be obtained by computing the following expectation:

\[ E_Y \left[ e^{\frac{\sigma_{dM}^2 R_{dM}}{R_{dM}}} - 1 \right] = \int_{0}^{\infty} \left( e^{\frac{\sigma_{dM}^2 R_{dM}}{R_{dM}}} - 1 \right) f_Y(y) \, dy \]

(30)

where $f_Y(y)$ is the pdf of $\sum_{j=1}^{J(0)} (1/y^{(j)})$ in (27).

We should find the pdf of $y^{(j)}$ to obtain $f_Y(y)$. Let us define the pdf of random variable $y^{(j)}$ and $\theta^{(j)}(=(1/y^{(j)}))$ as $f_{Y^{(j)}}(y)$ and $f_{\theta^{(j)}}(t)$, respectively. Since $y^{(j)}$ is a lognormal random variable with mean $(1/(\lambda^{(j)})E[\ln(Y/S_0)]+(1/\lambda^{(j)})\) and standard deviation $(1/\lambda^{(j)})\)Var[(\ln(Y/S_0)]$, by simple random variable transformation, the pdf of $\theta^{(j)}(=(1/y^{(j)}))$, $f_{\theta^{(j)}}(t)$, is given by

\[ f_{\theta^{(j)}}(t) = \frac{1}{t^2} f_{Y^{(j)}} \left( \frac{1}{t} \right) \].

Finally, $f_Y(y)$ can be obtained from convolutions of $f_{\theta^{(j)}}(t)$, $j = 1, 2, \ldots, J(0)$ as follows:

\[ f_Y(y) = f_{\theta^{(1)}}(t) \otimes f_{\theta^{(2)}}(t) \otimes \cdots \otimes f_{\theta^{(J(0))}}(t) \].

(32)

Now, the Erlang capacity is a set of values \{\lambda_d/\mu_{dM}, \lambda_{d1}/\mu_{d1}, \lambda_{dM}/\mu_{dM} \} that keep the Chernoff bound on the outage probability in (29) at a target level, typically 0.01. The model provides an upper bound on the achievable capacity of the forward link for a CDMA system with multirate sources.

### IV. Numerical Results

The numerical examples of the forward-link Erlang capacity are presented in this section. Particularly, we show the forward-link Erlang capacity of the third-generation CDMA system (cdma2000, UTRA) in various environments. The system can support any Erlang set that keeps the outage probability in (23) and (29) at a target level. When there are many multirate traffic sources, there will be many sets that keep the outage probability at a target level. Hence the peak Erlang capacity can be determined in a multidimensional surface. As the number of multirate sources increases, the dimension of surface for peak Erlang capacity is increased.

We show the capacity when there are at most three different multirate sources, and this will be sufficient to understand how the capacity is determined under the multirate traffic sources.

For the numerical examples, path-loss exponent $l = 4$, target outage probability of 0.01, soft handoff fraction $g = 0.3$, and a fraction of BS power assigned to traffic channels $\psi = 0.8$ are assumed. We consider two types of data services: a Web service and a real-time service. For the Web service, the average file size transmitted during a burst is assumed to be 13.9 Kbytes, and the mean OFF period $E[T_{\text{off}}]$ is 10.5 s [12]. For the real-time service, the data activity is assumed to be unity because it requires continuous transmission.

In Fig. 3, a forward-link Erlang capacity of the cdma2000 (chip rate $= 1.2288$ Mc/s) is shown for multipath environment, standard deviation of lognormal shadow fading 8 dB, forward-link power control error 2.5 dB, and power factor 0.4. We consider only three kinds of multirate traffic sources:

1. 9.6-kb/s voice service with the required $E_b/N_0$ of 4 dB and the voice activity factor 0.4;
2) 76.8-kb/s real-time data service with the required $E_b/N_t$ of 3 dB;
3) 153.6-kb/s Web browsing service with the average required $E_b/N_t$ of 3 dB.

Typically, the required $E_b/N_t$ depends on the system operating conditions, including vehicular speed and link level parameters. The Erlang capacity in this figure is determined by the three-dimensional surface. The system can support any Erlang sets below the three-dimensional surface. It is shown that the system can support more calls of 153-kb/s Web service than those of 76.8-kb/s real-time service because the capacity depends on not only the data rate of traffic sources but also the traffic pattern reflected in the activity factor. The average interference level with low activity factor will be smaller than that with high activity factor.

In Fig. 4, a forward-link Erlang capacity is shown for the cdma2000 (chip rate = 1.2288 Mc/s) with multipath environment, and three rake receivers, i.e., $J'(0) = 3$ with power por-
Fig. 6. Forward-link Erlang capacity bounds of cdma2000 (chip rate $1.2288$ Mc/s) with multipath components.

Fig. 7. Forward-link Erlang capacity of cdma2000 (chip rate $3.6864$ Mc/s) with no multipath component.

tions for the three-tap rake receivers are $(0.8, 0.1, 0.05)$, respectively. Other parameters assumed in this figure are the same as those of Fig. 3. It is shown that the capacity for the case with multipath components is reduced compared to the case without multipath components. Under the multipath environment, the orthogonality among code channels cannot be maintained, and this eventually leads to the capacity decrease.

Fig. 5 shows the impact of power control error on the forward-link capacity under the multipath environments. This is for the cdma2000 (chip rate $1.2288$ Mc/s), as in Figs. 3 and 4. The parameters assumed in this figure are the same as those of Fig. 4 except that there are only two kinds of multi-rate traffic sources: 9.6-kb/s voice and 76.8-kb/s real-time services. The capacity is gradually reduced with the power control error. The power control error increases variance of interference power level, which leads to the increase of outage probability. It is confirmed that the power control error of the source with high data rate and high activity factor can dramatically decrease the capacity.

In Fig. 6, we show forward-link Erlang capacity bounds of the cdma2000 (chip rate $1.2288$ Mc/s) under various multipath environments. The power portions of three-tap rake receivers for case 1, case 2, and case 3 are given by $(0.9, 0.05, 0.05)$, $(0.8, 0.1, 0.01)$, and $(0.75, 0.2, 0.01)$, respectively. The power control
error is set to be 2.5 dB, and other conditions are the same as those of Figs. 4 and 5. The two kinds of multirate sources are considered: 9.6-kb/s voice and 153.6-kb/s Web browsing services. As the power of the strongest path increases, the capacity is gradually increased. The orthogonality among code channels is degraded as the received power is distributed in the multipath components.

In Fig. 7, forward-link Erlang capacity of the cdma2000 chip rate = 3.6864 Mc/s is shown for no multipath components, standard deviation of lognormal shadow fading 8 dB, power control error of 2.5 dB, and forward link power factor 0.4. Three kinds of multirate traffic sources are considered:

1) 9.6-kb/s voice service with the required $E_b/N_0$ of 4 dB and the voice activity factor of 0.4;
2) 153.6-kb/s real-time data service with the required $E_b/N_0$ of 3 dB;
3) 307.2-kb/s Web browsing service with the required $E_b/N_0$ of 3 dB.

The system can support any Erlang sets below the three-dimensional surface. The capacity is proportional to the spreading
bandwidth, but it is not linearly proportional because the outage probability is not a linear function of spreading bandwidth.

In Fig. 8, a forward-link Erlang capacity bound of the cdma2000 (chip rate $= 3.6864$ Mc/s) is shown for three-tap rake receivers, i.e., $J'(0) = 3$ with power portions (0.75, 0.2, 0.01), power control error of 2.0 dB, and forward-link power factor 0.4. Three kinds of multirate traffic sources are considered:

1) 9.6-kb/s voice service with the required $E_b/N_t$ of 4 dB and the voice activity factor of 0.4;
2) 153.6-kb/s real-time data service with the required $E_b/N_t$ of 5 dB;
3) 307.2-kb/s Web browsing service with the required $E_b/N_t$ of 3 dB.

Compared to Fig. 7, the capacity is decreased, though the power control error is reduced from 2.5 to 2.0 dB, because the orthogonality among code channels is not guaranteed due to the multipath components make.

In Fig. 9, we show the impact of power control error on the forward link capacity of the cdma2000 (chip rate $= 3.6864$ Mc/s) under multipath environment. The conditions assumed in this figure are the same as those of Fig. 8, except that there are only two kinds of multirate traffic sources: 9.6-kb/s voice and 153.6-kb/s real-time services. As in Fig. 5, it is confirmed that the power control error has a negative influence on the forward-link capacity.

In Fig. 10, forward-link Erlang capacity of the cdma2000 (chip rate $= 3.6864$ Mc/s) is shown for various multipath environments. The setting of tap coefficient for the three-tap rake receiver is the same as in Fig. 6 for case 1, case 2, and case 3. Two kinds of mixed sources are considered: 9.6-kb/s voice and 153.6-kb/s Web browsing services. The power control error is set to be 2.0 dB, and other conditions are the same as those of Figs. 8 and 9. It is confirmed that the capacity is increased with the power of strongest path.

In Fig. 11, forward-link Erlang capacity of the UTRA (chip rate $= 3.84$ Mc/s) is shown for no multipath components, standard deviation of lognormal shadow fading 10 dB, forward-link power control error 2.0 dB, and forward-link power factor 0.4. Three kinds of multirate traffic sources are considered:

1) 8-kb/s voice service with the required $E_b/N_t$ of 5 dB and the voice activity factor 0.5;
2) 144-kb/s real-time data service with the required $E_b/N_t$ of 3 dB;
3) 384-kb/s Web browsing service with the required $E_b/N_t$ of 3 dB.

The data rates supported by UTRA are not the same as those of cdma2000, so that the direct comparison with Fig. 7, which shows the capacity of cdma2000 (chip rate $= 3.6864$ Mc/s), is not available. The capacity gap between these two systems depends on the conditions such as power control error, shadowing fading, multipath, etc.

V. CONCLUSION

In this paper, we derived the outage probability and the Erlang capacity of forward link of the CDMA system with mixed multirate sources. By introducing the forward-link power factor, the forward-link outage probability and Erlang capacity were derived in a closed form. The forward-link capacity was analyzed in term of various parameters, such as the number of multipaths, the number of rake fingers in a mobile station, power control error, and impact of soft handoff. We showed that the capacity of a CDMA system with multirate sources could be effectively depicted in the multidimensional surface through the numerical examples. We showed the forward-link capacity of the specific third-generation CDMA systems (cdma2000, UTRA) by indicating the chip rate under various conditions. From the numerical results, it was shown that the power control error has a negative influence on the forward-link capacity, and the capacity is
increased with the power of strongest path under the multipath environments.

In the future mobile communication systems, it is expected that a forward link can be a limiting link because prospective services are likely to require higher data rate in the forward link than in the reverse link. The results in this paper can be applied to overall system design of a CDMA system with multimedia services in future mobile communication systems.

**APPENDIX A**

**DISTRIBUTION OF $I_{oc}/S_0$**

To find distribution of $I_{oc}/S_0$, the distribution for the sum of 11 random variables in the numerator of (8) must be found. To do this, we should find the distribution of the function in the form of $y = c \cdot x$, where $c$ is a constant and $x$ is a lognormal random variable with mean dB value $m_x$ and standard deviation of dB value $\sigma_x$. By simple random variable transformation, it is found that $y$ is again a lognormal random with mean dB value $m_y = m_x + 10 \log c$ and the same standard deviation of dB value $\sigma_x$.

The probability density function of $y$ is given by

$$f(y) = \frac{1}{\beta \sqrt{2\pi} \sigma_x y} \exp \left[ -\frac{(10\log y - 10\log c - m_x)^2}{2 \cdot \sigma_x^2} \right] \tag{A1}$$

where $\beta = \ln(10)/10$ is a unit conversion constant.

Let the sum of 11 random variables in numerator in (8b) be $S$. Then, $S$ can be approximated as a lognormal random variable since the sum of lognormal random variables can be approximately lognormal [15]. The mean and variance of $S$ can be obtained as

$$E[S] = 2 \cdot f(m_x, \sigma_x) + 3 \cdot f(m_x + \ln(2)^2, \sigma_x) + 6 \cdot f(m_x + \ln(2.633)^2, \sigma_x) \tag{A2}$$

$$\text{Var}[S] = 2 \cdot h(m_x, \sigma_x) + 3 \cdot h(m_x + \ln(2)^2, \sigma_x) + 6 \cdot h(m_x + \ln(2.633)^2, \sigma_x) \tag{A3}$$

where $m_x$ and $\sigma_x$ are the mean dB value and the standard deviation of dB value of a lognormally distributed random variables $X_k$ for $k = 1, 2, \ldots, 11$, respectively, $f(x, b) = \exp\{b \alpha + \beta b^2/2\}$, and $h(x, b) = \exp\{2b\alpha + \beta b^2\} \cdot (\exp\{\beta b^2\} - 1)$.

The mean dB value and the standard deviation of dB value of a lognormal random variable, $\bar{S} = 10 \log S$, can be computed by

$$\text{Var}[\bar{S}] = \sigma_s^2 = \frac{1}{\beta^2} \ln \left( \frac{\text{Var}[S]}{E[S]^2} + 1 \right) \tag{A4}$$

$$E[\bar{S}] = m_s = \frac{\ln[E[S]]}{\beta} - 0.5 \cdot \beta \cdot \sigma_s^2 \tag{A5}$$

Since the $S$ is lognormally distributed random variable with mean dB value $m_s$ and standard deviation of dB value $\sigma_s$, and the $X_0$ is also a lognormal random variable with mean dB value $m_0$ and standard deviation of dB value $\sigma_0$, the $S$ and $X_0$ can be, respectively, represented as

$$S = 10^{m_s + X_s/10} \tag{A6}$$

$$X_0 = 10^{m_0 + X_0/10} \tag{A7}$$

where $X_s$ and $X_0$ are Gaussian random variables with zero mean and standard deviation $\sigma_s$ and $\sigma_0$, respectively.

Then, from (A6) and (A7), $(I_{oc}/S_0)$ can be obtained as

$$\frac{I_{oc}}{S_0} = \frac{S}{X_0} = 10^{m_s - m_0 + X_s - X_0/10} \tag{A8}$$

From (A8), we can find that $(I_{oc}/S_0)$ is a lognormal random variable with mean dB value $m_{is} = m_s - m_0$ and standard deviation of dB value $\sqrt{(\sigma_s^2 + \sigma_0^2)}$ because $X_s - X_0$ is a Gaussian random variable with zero mean and standard deviation $\sqrt{(\sigma_s^2 + \sigma_0^2)}$.

**APPENDIX B**

**DERIVATION OF AVERAGE FORWARD LINK POWER FACTOR**

The relative power for the $i$th MS is proportional to $I_{oc}/S_0$ from (13)–(15). For the case that the users are uniformly distributed, the ratio of interference from the $j$th adjacent BS to the received power from home BS is given by (B1a)–(B1c), shown at the bottom of the page, where $f_{r, \theta}(x, y)$ is the pdf of locations of MSs, which are uniformly distributed in the cell, and given by

$$f_{r, \theta}(x, y) = \frac{2}{R^2 \theta_s}, \quad 0 \leq x \leq R, \quad 0 \leq y \leq \theta_s \tag{B2}$$

where $R$ is the radius of a cell, $\theta_s$ is the angle of a sector, and $\theta_s = 2\pi$ for the unsectorized cell.

$I_{oc}/S_0$ for the case that users are uniformly distributed in the cell is $\eta$ times that of the worst case that all users are at the cell boundary. Hence, from (B1a)–(B1c), $\eta$ is given by (B3),

$$\frac{I_{oc,j}}{S_0} = \int_0^{\theta_s} \int_0^R x \cdot \frac{I_{oc,j}(x)}{S_0(x)} \cdot f_{r, \theta}(x, y) \, dx \, dy \tag{B1a}$$

$$= \int_0^{\theta_s} \int_0^R x \cdot \frac{I_{oc,j}(R)}{S_0(R)} \cdot \left( \frac{R \sqrt{R^2 + x^2 - 2Rx \cos(\theta_j - y)}}{x \sqrt{R^2 + R^2 - 2Rx \cos(\theta_j)}} \right)^{-\gamma} \frac{2}{R^2 \theta_s} \, dx \, dy \tag{B1b}$$

$$= \frac{I_{oc,j}(R)}{S_0(R)} \int_0^{\theta_s} \int_0^R x \cdot \left( \frac{R \sqrt{R^2 + x^2 - 2Rx \cos(\theta_j - y)}}{x \sqrt{R^2 + R^2 - 2Rx \cos(\theta_j)}} \right)^{-\gamma} \frac{2}{R^2 \theta_s} \, dx \, dy \tag{B1c}$$

$$= \frac{I_{oc,j}(R)}{S_0(R)} \int_0^{\theta_s} \int_0^R x \cdot \left( \frac{R \sqrt{R^2 + x^2 - 2Rx \cos(\theta_j - y)}}{x \sqrt{R^2 + R^2 - 2Rx \cos(\theta_j)}} \right)^{-\gamma} \frac{2}{R^2 \theta_s} \, dx \, dy \tag{B1c}$$
shown at the bottom of the page. In Fig. 12, we show the cell configuration. The $(r, \theta)$ coordination is given as follows.

Let the $(r, \theta)$ coordination of the adjacent $j$th BS be $(R_j, \theta_j)$. In the derivation of forward link power factor, the following should be noted.

1) The average forward-link traffic channel power is about $\eta$ times the traffic channel power needed to serve all the MSs at the cell boundary.

2) The typical value of $\eta$ is 0.4 when the path-loss exponent is four. However, in a real environment, it should be generally greater than 0.4 because it is the average value, that is, the forward-link power control error can vary its value.

3) We used an average forward-link power factor to take into account the fact that the MSs are uniformly distributed in the cell, not at the cell boundary. We extended the average forward power factor into the multiple cell environment. The average forward-link power factor was introduced in [17], but it is derived only for the single cell environment as follows:

\[
\overline{P_{\text{traffic}}} = \int_0^\theta \int_0^R x \cdot P_{\text{traffic}}(x) \cdot f_{r,\theta}(x, y) \, dx \, dy
\]
\[
= \int_0^R x \cdot P_{\text{traffic}}(R) \cdot \left(\frac{x}{R}\right)^\gamma \frac{2}{R^2} \, dx
\]
\[
= P_{\text{traffic}}(R) \int_0^1 2t^{\gamma+1} \, dt
\]
\[
= P_{\text{traffic}}(R) \cdot \frac{2}{\gamma+2}
\]
\[
= \eta \cdot P_{\text{traffic}}(R)
\]

where $\overline{P_{\text{traffic}}}$ is average traffic power needed to serve the uniformly distributed MSs and $P_{\text{traffic}}(R)$ is the traffic power needed to serve all the MSs at the cell boundary.

**APPENDIX C**

**CHERNOFF BOUND ON OUTAGE PROBABILITY**

The Chernoff bound on the outage probability in (28) can be represented by (C1a) and (C1b), shown at the bottom of the page, where $E_{K_{\omega}}$, $E_Y$, and $E_\rho(\cdot)$ denote the expectations on random variable $K_{\omega}$, $Y$, and $\rho_q(\cdot)$, respectively.

\[
\eta = \sum_j \int_0^{\theta_j} \int_0^R x \cdot \left( \frac{R \sqrt{R_j^2 + x^2 - 2R_jx\cos(\theta_j - y)}}{x \sqrt{R_j^2 + R^2 - 2R_jR\cos|\theta_j|}} \right)^{-\gamma} \frac{2}{R^2\theta_j} \, dx \, dy. \tag{B3}
\]

\[
P_{\text{out}} < \min_{\alpha > 0} E_{K_{\omega}} \left[ \prod_{j=1}^{K_{\omega}} E_Y \left[ E_{\rho_{\omega}} \left[ \exp \left\{ s \cdot \frac{\gamma_{\omega} R_{\omega}}{W} Y \right\} \right] \right] \right] \nonumber
\]
\[
\cdot E_{K_{d_h}} \left[ \prod_{j=1}^{K_{d_h}} E_Y \left[ E_{\rho_{d_h}} \left[ \exp \left\{ s \cdot \frac{\gamma_{d_h} R_{d_h}}{W} Y \right\} \right] \right] \right] \nonumber
\]
\[
E_{K_{d_M}} \left[ \prod_{j=1}^{K_{d_M}} E_Y \left[ E_{\rho_{d_M}} \left[ \exp \left\{ s \cdot \frac{\gamma_{d_M} R_{d_M}}{W} Y \right\} \right] \right] \right] \cdot e^{-s\tilde{\beta}} \tag{C1a}
\]
\[
= \min_{\alpha > 0} E_{K_{\omega}} \left[ \left\{ E_Y \left[ e^{\frac{\gamma_{\omega} R_{\omega}}{W} \cdot \rho_{\omega} + (1 - \rho_{\omega})} \right] \right\]^{K_{\omega}} \right] \nonumber
\]
\[
\cdot E_{K_{d_h}} \left[ \left\{ E_Y \left[ e^{\frac{\gamma_{d_h} R_{d_h}}{W} \cdot \rho_{d_h} + (1 - \rho_{d_h})} \right] \right\]^{K_{d_h}} \right] \nonumber
\]
\[
E_{K_{d_M}} \left[ \left\{ E_Y \left[ e^{\frac{\gamma_{d_M} R_{d_M}}{W} \cdot \rho_{d_M} + (1 - \rho_{d_M})} \right] \right\]^{K_{d_M}} \right] \cdot e^{-s\tilde{\beta}} \tag{C1b}
\]
Since $K_{21}, K_{22}, \ldots, K_{2M}$ have Poisson distribution, the expectations on $K_{21}, K_{22}, \ldots, K_{2M}$ are derived as

$$
E_{K_2} = \sum_{K_{2i}=0}^{\infty} \frac{1}{K_{2i}!} \left( \frac{\lambda_v}{\rho_v} \cdot E_Y \left[ e^{\frac{\rho_v}{\lambda_v}} - 1 \right] \right)^{K_{2i}} K_v
$$

Where $\lambda_v$ and $\rho_v$ denote the expectations on the random variable $Y$. Hence, the Chernoff bound on the outage probability in (27) is obtained as

$$
P_{out} < \min_{\delta > 0} e^{\frac{\rho_v}{\lambda_v}} \sum_{K_{2i}=0}^{\infty} \frac{1}{K_{2i}!} \left( \frac{\lambda_v}{\rho_v} \cdot E_Y \left[ e^{\frac{\rho_v}{\lambda_v}} - 1 \right] \right)^{K_{2i}} K_v
$$

Where $E_Y$ denotes the expectations on the random variable $Y$.