Multistage Detection in Asynchronous Code-Division Multiple-Access Communications

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Abstract—A multiuser detection strategy for coherent demodulation in an asynchronous code-division multiple-access system is developed and analyzed. The resulting detectors process the sufficient statistics via a multistage algorithm. This algorithm is based on a successive multiple-access interference annihilation scheme. An efficient real-time implementation of the multistage algorithm with a fixed decoding delay is obtained and it is shown to require a computational complexity which is linear in the number of users \( K \). Hence, the multistage detector contrasts the optimum demodulator, which is based on a dynamic programming algorithm; has a variable decoding delay; and a software complexity per symbol that is exponential in \( K \) [18]. Further, an exact expression for the probability of error is obtained for the two-stage detector. The probability of error computations show that the two-stage receiver is particularly well suited for “near-far” situations. In fact, performance approaches that of single-user communications as the interfering signals become weaker. The near-far problem is therefore alleviated. Further, significant performance gains over the conventional receiver are obtained even for relatively high bandwidth efficiency situations.

I. INTRODUCTION

In a code-division multiple-access (CDMA) system, several users simultaneously transmit information over a common channel using preassigned code waveforms. The receiver is equipped with a knowledge of the codes of some or all of the users. It is then required to demodulate the information symbol sequences of these users, upon reception of the sum of the transmitted signals of all the users in the presence of additive noise. Examples of such situations arise in a variety of communication systems such as satellite communications, radio networks and other multipoint-to-multipoint multiple-access networks.

In a majority of the CDMA systems of practical importance, the users transmit information independently. Therefore the transmitted signals of different users arrive asynchronously at the receiver. Since their relative time delays are arbitrary, it is inevitable that the cross-correlations between signals of different users are nonzero. Low cross-correlations among code waveforms for all relative time delays are obtained by the design of a set of complex code waveforms at the expense of an increased bandwidth [15]. Since bandwidth is a valuable resource, the problem of interest is to be able to accommodate as many users as can be reliably demodulated for a given bandwidth.

The conventional approach to multistage demodulation is to demodulate each user’s signal as if it were the only one present. This receiver consists of a bank of filters matched to each user’s signal waveform in corresponding time and phase synchronism as in Fig. 1. The \( k \)th user’s decision statistic consists of the desired signal component, additive noise, and multiple-access interference due to the cross-correlations of the \( k \)th user’s signal with the signals from the other users. This receiver makes decisions by comparing the statistic to a threshold and has the advantage of simplicity. It also lends itself to expedient modification for decentralized reception. Extensive analysis of the conventional detector for spread-spectrum systems has been undertaken over the past few years. Different methods for computation of bounds and approximations of the probability of error can be found in [2], [5], [7], [9], [14] and the references therein. These studies have revealed the usefulness of a careful design of the signal constellation wherein acceptable performance from the conventional receiver is possible due to low cross-correlations between signals of different users. However, as the number of users in a system of fixed bandwidth grows or as the relative powers of the interfering signals become large (the “near-far” problem), severe performance degradation of the suboptimum systems is observed even for relatively low bandwidth efficiencies [1], [2]. This effect has been succinctly characterized by Verdú in [19], in terms of asymptotic efficiency, a measure that quantifies the multiple-access limitation of a detector.

The optimum multiuser demodulator can be obtained based on a maximum likelihood sequence detection formulation. If all the information matrices are equiprobable, the maximum likelihood sequence detector is obtained by maximizing the joint \textit{a posteriori} probability

\[ p(B | \{r(t); t \in R \}) \]

(1.1)

where \( r(t) \) denotes the received signal, \( B \) represents the information bit matrix, the \((k, t)\)th element of which is the \( k \)th bit of the \( k \)th user denoted as \( b_{k,t} \). If the packet length of the information bit sequence of each user is \( 2^{P+1} \) and the number of users in the system is \( K \), then an exhaustive maximization of the joint \textit{a posteriori} probability needs its computation for each of the \( 2^{2^{P+1}+1} \) possible values of \( B \). Such a brute-force maximization is practically useless. However, a significant computational gain over this exhaustive scheme was obtained by Verdú in [18] by exploiting the additive decomposability of the
log-likelihood function, paving the way for its maximization by a forward dynamic programming algorithm of the Viterbi type. Although the computational and storage complexity of this efficient algorithm was independent of the packet length, it was however shown to depend exponentially on the number of users. It was also shown that the multistage optimum detection problem is NP-hard. In addition to requiring intensive software complexity, the maximum likelihood solution also has a variable decoding delay.

From earlier discussion it is clear that despite its simplicity, the conventional receiver has limited use for CDMA systems in the desirable range of bandwidth efficiencies. In addition, its vulnerability to near-far effects is considerable. On the other hand, the optimum receiver is computationally too intensive, and has a variable decoding delay which is unacceptable in many applications. There is hence a need for suboptimum receivers which perform reliably in higher bandwidth efficiency situations and are robust to near-far effects with a reasonable computational complexity to ensure their practical implementation.

In this paper, we propose and analyze detectors based on a multistage detection strategy of successive multiple-access interference rejection. The resulting detectors process the sufficient statistics via a multistage algorithm. An efficient real-time implementation of the multistage algorithm with a fixed decoding delay is obtained and it is shown to require a computational complexity/symbol which is linear in the number of users \( K \), in obvious contrast to the optimum demodulator, which has a software complexity per symbol that is exponential in \( K \). It is also shown that in a system with well-designed code waveforms, the performance of the multistage detector closely tracks that of the optimum receiver. In most cases of practical interest, the proposed receiver achieves significant performance gains over the conventional receiver. As in the case of the optimum detector, the proposed multistage receiver requires a knowledge of the signal strengths for its implementation. In some practical situations where exact values of the energies are not available, estimates can be used instead [17]. However, for the analysis presented in this paper, an exact knowledge of energies is assumed.

The rest of the paper is organized as follows. A general multistage CDMA communication model is described in Section II. While the proposed multistage multiple-access interference rejection scheme is valid for the general CDMA system, we will for most part of the paper be concerned with the analysis of the more specific BPSK-CDMA system which we shall also describe in this section. Section III deals with the development of the multistage algorithm and some insight into the resulting detector is also presented. In Section IV, an exact expression for the probability of error is obtained for the two-stage detector. In Section V, we will present numerical results comparing the two-stage, the conventional, and the single-user demodulators (i.e., matched filter operating in the absence of interfering signals) and thereby illustrate achievable gains in bandwidth efficiency and near-far immunity. In that same section, we also analyze the proposed receiver for a direct-sequence spread-spectrum multiple-access system.

II. SYSTEM DESCRIPTION

Let us assume that there are \( K \) transmitters in the system, and each transmitter employs an \( S \)-ary signal set derived from the \( S \) code waveforms assigned to the corresponding user. We will now introduce some notation to describe the received signal waveform. Suppose \( S_k = \{s_{k, 0}, s_{k, 1}, \ldots, s_{k, S-1}\} \) is the \( S \)-ary signal set of the \( k \)th user, where each signal is time-limited to \([0, T]\), \( T \) being the symbol duration assumed to be equal for all users. Denote the diagonal energy matrix \( E \) so that \( \text{diag}(E) = [E_1, E_2, \ldots, E_K] \) where \( E_k \) is the received energy of the \( k \)th user's signal which is assumed to be independent of the transmitted symbol. Let \( P \) be the set of time indices \([0, \ldots, P]\) with \( P > 1 \) being the packet length, and let \( S \) be a set of \( S \) indexes \([0, \ldots, S-1]\). Assume that the \( k \)th user transmits its \( j \)th symbol by transmitting \( s_{k,j} \). Representing the sequence of information symbols of the \( k \)th user by a map \( b_k : I \to S_k \), we have that if \( b_k(i) = j \), the \( j \)th symbol of the \( k \)th user is \( i \). Define \( B_k \) to be a \( K \times (2P + 1) \) information matrix, representing the maps \( b_1, b_2, \ldots, b_K \), so that the \((k,i)\)th element of \( B \) is equal to \( b_k(i) \). Hence, the received signal is modeled as being the sum of \( K \) signals denoted by \( s(t,B) \) and additive white Gaussian noise \( n_t \) with spectral density \( N_0/2 \). The received process is therefore given as

\[
\begin{align*}
\mathbf{r}(t) &= s(t,B) + n_t, t \in R
\end{align*}
\]

where

\[
\begin{align*}
s(t,B) &= \sum_{k=1}^{K} s_k(t) - \tau_k, (t \in R),
\end{align*}
\]

so that \( \tau_k \in [0, T] \) represents the time delay of the \( k \)th user. Each value of the matrix \( B \) corresponds to the specification of packets transmitted by all the users and so there are \( S^K (2P + 1) \) possible maps \( B \). The centralized demodulator that we propose in this paper is general enough to be valid for the CDMA situation just described, which we will call the general CDMA system.

However, for the sake of simplicity, we will often restrict ourselves to a more specific system wherein each user is assigned one code waveform. Denote the modulating signal of the \( k \)th user as \( s_k(t) \). The \( k \)th user then antipodally modulates this signal to transmit information bits. In the general CDMA system, this corresponds to \( S = 2, b_k(t) \in \{0, 1\} \), \( s_k(0) = s_k(t) \) and \( s_k(t) = -s_k(t) \), so that the transmitted signal is

\[
\begin{align*}
s(t,B) &= \sum_{k=1}^{K} (-1)^{b_k(t)} s_k(t - iT - \tau_k)
\end{align*}
\]

where \( b_k(t) = (-1)^{b_k(t)} \in \{-1, 1\} \). This system will be referred to as the BPSK-CDMA system.

The essential system parameters are the signal cross-correlations and their relative energies. Following the notation in [16] let us define the \( K \times K \) cross-correlation matrices \( H(t) \) such that the \((k,i)\)th element is

\[
\begin{align*}
h_{ki}(t) &= \int_{-\infty}^{\infty} s_k(t - \tau) s_i(t + iT - \tau) \, dt.
\end{align*}
\]

For simplicity and without loss of generality, let us assume an ordering on the time delays \( \tau_k \) such that \( 0 \leq \tau_1 \leq \tau_2 \leq \cdots \leq \tau_K < T \). Since the modulating signals are time-limited, \( H(t) = 0 \forall |t| > 1 \) and \( H(-i) = H^t(i) \). Note that \( H(1) \) is an upper triangular matrix with a zero diagonal. In the next section, the multiple-access communication system described above is considered for multistage demodulation.

III. THE MULTISTAGE DETECTOR

In this section, we present a development of the multistage scheme for the multistage demodulation problem specified in the previous section. First the sufficient statistics for optimum demodulation are described. In Section III-A, we will derive the multistage multiple-access rejecting detector for the BPSK-CDMA system first and then extend it for the general CDMA system and present the multistage detector in algorithmic form. In Section III-B, we will address some important implementation issues.

Consider the maximum likelihood sequence detection formulation for the problem. The most likely information matrix is chosen as that which maximizes the probability in (1.1) or equivalently, the log-likelihood function [16]

\[
\begin{align*}
\mathbf{L}(B, r(t)) &= 2\int s(t, B)r(t) \, dt - \int s^2(t, B) \, dt.
\end{align*}
\]

The above expression shows that the sufficient statistics are the out-
puts of a bank of $K$ matched filters matched to the modulating signal of each user, sampled in corresponding time synchronism. The output of the $k$th matched filter sampled at the end of the $i$th time interval is given as

\[ z_i^{(k)}(0) = \int_{-\infty}^{\infty} r(t) \chi_k(t + iT - r_i) \, dt \quad (3.5) \]

\[ = \eta_0^{(k)} + \sum_{i=1}^{K} h_{ki}(1)b_i^{(i-1)} + \sum_{i=1}^{K} h_{ki}(0)b_i^{(i)} + \sum_{i=1}^{k-1} h_{ki}(-1)b_i^{(i+1)} \quad (3.6) \]

where $\eta_0^{(k)}$ is the component of the statistic due to the additive channel noise. In vector notation, letting $z_i^{(0)}(0)$ denote $[z_i^{(0)}(0), z_i^{(1)}(0), \ldots, z_i^{(K)}(0)]^T$, we have

\[ z_i^{(0)}(0) = \eta_0^{(0)} + H(1)b_i^{(i-1)} + H(0)b_i^{(i)} + H(-1)b_i^{(i+1)} \quad (3.7) \]

where $b_i^{(j)}$ is the $j$th column of $B$. The set of sufficient statistics for the maximum likelihood sequence detection of the sequence $\{b_i^{(j)}; \forall i\}$ is therefore $[z_i^{(0)}(0), \forall i]$. In fact, this set of sufficient statistics is minimally sufficient for the demodulation of any one bit [18].

A. Derivation

The multitap suboptimum solution to the maximization of $L[B, r(t)]$ is now developed. Consider the demodulation of the $i$th bit of the $k$th user. Assume that we are in the $(m+1)$th stage. Let the $m$th stage estimates of bits $b_i^{(j)}$ be denoted as $b_i^{(j)}(m)$ for all $i$ and for all $j$. We propose the $(m+1)$st stage estimate of $b_i^{(j)}$ as being

\[ b_i^{(j)}(m+1) = \arg \max_{b_i^{(j)}(m+1)} \left[ L[B, r(t)] \right] \quad (3.8) \]

\[ \text{Note that the maximization is performed by setting } b_i^{(j)}(m+1) \text{ to be equal to its } m \text{th stage estimate for all } i \text{ and for all } j \text{ but not } i = k \text{ and } j = i \text{ simultaneously. From (2.2), (3.5), and (3.4), an additive decomposition of the log-likelihood function can be obtained as} \]

\[ L[B, r(t)] = \sum_p b_i^{(p)}, 2z_i^{(p)}(0) - \sum_p \sum_{i=1}^{K} b_i^{(i)}, 2z_i^{(i)}(0) \quad (3.9) \]

where $\chi, x$ denotes the inner product between the vectors $x$ and $y$. Substituting from (3.9) into (3.8) and retaining only terms in the summation that involve $b_i^{(j)}$, we have

\[ b_i^{(j)}(m+1) = \arg \max_{b_i^{(j)}(m+1)} \left\{ \sum_{i=1}^{K} \sum_{p=1}^{K} b_i^{(j)}(m), z_i^{(j)}(0) \quad (3.10) \right. \]

\[ - \quad H(0)b_i^{(m)} + H(1)b_i^{(i-1)} \quad (3.11) \]

First each inner product is expanded, the terms that depend on $b_i^{(0)}$ are then retained. Substituting the $m$th stage estimates for all bits except $b_i^{(j)}$ and noting the properties of $H(.)$, we have the following expression for the estimate:

\[ b_i^{(j)}(m+1) = \text{sgn}[z_i^{(j)}(m)] \quad (3.12) \]

where

\[ z_i^{(j)}(m) = z_i^{(j)}(0) - \sum_{i=1}^{K} h_{ki}(1)b_i^{(i-1)}(m) \]

\[ - \sum_{i=1}^{K} h_{ki}(0)b_i^{(i)}(m) - \sum_{i=1}^{K} h_{ki}(-1)b_i^{(i+1)}(m). \quad (3.11) \]

The above result has a simple interpretation.\cite{Note 3} Note from (3.6), (3.10), and (3.11) that the $(m+1)$st stage decision statistic is obtained by subtracting the estimate of the multiple-access interference (denoted by $I_i^{(d)}(m)$ in Fig. 3) which is reconstructed using the $m$th stage estimates of the information bits.\cite{Note 2} Substituting for $z_i^{(j)}(0)$ from (3.6) into (3.11), we have

\[ z_i^{(j)}(m) = \eta_0^{(j)} + E_i b_i^{(j)} + \sum_{i=1}^{K} h_{ki}(1)b_i^{(i-1)}(m) \]

\[ + \sum_{i=1}^{K} h_{ki}(0)b_i^{(i)}(m) + \sum_{i=1}^{K} h_{ki}(-1)b_i^{(i+1)}(m) \]

\[ + \sum_{j=1}^{K} h_{kj}(1)b_i^{(i-1)}(m) \quad (3.12) \]

\[ \text{where } \eta_0^{(j)} + E_i b_i^{(j)} \text{ is equal to } 2b_i^{(j)} \text{ or zero depending on whether } b_i^{(j)}(m) \text{ is in error or not.} \]

We now seek to express the $(m+1)$st stage update of the estimate of $b_i^{(j)}$ using matrix notation. Performing the maximization of (3.8) and the $m$th stage estimate of the vector $b_i^{(j)}$, we have the $(m+1)$st-stage vector decision statistic written in matrix notation as

\[ z_i^{(j)}(m) = \eta_0^{(j)} + E_i b_i^{(j)} + H(0)b_i^{(i)} - H(1)b_i^{(i-1)} \quad (3.13) \]

\[ + \sum_{j=1}^{K} h_{kj}(1)b_i^{(i-1)}(m) - H(0)b_i^{(i)} - H(1)b_i^{(i+1)}(m) \]

\[ \text{where } d_i^{(j)}(m) \pm b_i^{(j)}(m). \]

The $(m+1)$st stage update of the iteration is given as

\[ b_i^{(j)}(m+1) = \text{sgn}[z_i^{(j)}(m)], \quad m \geq 1. \quad (3.14) \]

Having described the multitap solution, we now need to choose an initial estimate of the information bits denoted by $b_i^{(1)}$, \forall i. We choose the conventional detector for the first stage for reasons of conceptual simplicity in implementation and analysis, i.e.,

\[ b_i^{(1)} = \text{sgn}[z_i^{(1)}(0)], \quad \forall i. \quad (3.15) \]

This initial assignment implies that the proposed multitap detector aims at improving the conventional decision on the information bits. Other suboptimum initial conditions can be substituted depending on how much computation one is willing to do. It would seem important, as it would then be able to form an iterative solution, that a reasonably good initial estimate be selected. An exact quantification of this statement will be provided by the probability of error analysis in Section IV.

For the general CDMA problem, the multitap detector is similar to the one described above. It consists of a bank of $SK$ matched filters followed by a multitap algorithm. The detector in this case can be described in the following algorithmic form:

I) Obtain sufficient statistics (stage 0) from the received signal waveform.

II) Perform $M$ stages of processing the sufficient statistics where the $m$th ($m \geq 1$) stage processor acts on the statistics produced by the $(m-1)$st stage. The $m$th stage consists of the following procedure:

i) estimation of the unknown symbols from the $(m-1)$st stage statistics.\cite{Note 3} If $m = M$, stop; else, proceed with ii).

ii) reconstruction of the MA interference using the estimates obtained in step i) and subtraction of the reconstructed MA interference from the sufficient statistics to obtain the $m$th stage statistics.

\cite{1}The same result is obtained if one starts from the minimum probability of bit-error formulation [18] and proceeds in a manner analogous to the one followed here.

\cite{2}In [4] and [3], the theoretical implication of a somewhat similar idea has been studied, albeit without reference to any particular modulation or multiple-access technique. The reader is also referred to [20] and [11] for interference cancellation techniques for demodulation in a spread-spectrum multiple-access system.

\cite{3}The unknown symbols are estimated in a simple suboptimum fashion making use of the low cross-correlation properties among signals of different users.
Fig. 2. The multistage multiuser detector for the BPSK-CDMA system.

The optimum demodulation of each symbol requires the observation of the received signal for its entire duration. Hence, it is necessary that, for the M-stage detector performance to approach near-optimality as M increases, the time interval over which the demodulator would have to process the received signal for the demodulation of a certain symbol should increase. It is shown in the Appendix that this is indeed the case and further the effective time interval over which the M-stage receiver processes the observed signal for demodulation of a symbol is quantified. As the number of stages increases by one, the time interval over which the signal is effectively processed increases so as to accommodate 2(K − 1) more information symbols. In the Mth stage, the interference due to the information symbols that directly overlap with the symbol under consideration are rejected based on the (M − 1)st stage receiver’s estimates of these symbols. Each of the 2(K − 1) interfering symbols is estimated by the (M − 1)st stage receiver by rejecting the corresponding interference based on the symbol estimates from the (M − 2)nd stage of the M-stage algorithm and so on. It is clear that the multiple-access interference is rejected with a weighted emphasis. If the probability of error decreases as the number of stages increases, the maximum weight is assigned to the symbols that directly overlap with the symbol under consideration for demodulation.

B. Implementation

A realizable implementation of the multistage algorithm should never need a symbol estimate that has either not yet been made or was obtained but not stored long enough. An efficient implementation should store symbol estimates at various stages only as long as they are needed and avoid repetition of any processing.

A detector which efficiently incorporates the multistage algorithm for the BPSK-CDMA system is shown in Figs. 2 and 3. Fig. 2 shows a bank of K matched filters followed by a set of K M-stage processors. The detailed implementation of the kth such processor is shown in Fig. 3. The signal flow is depicted by a snap-shot of the receiver at time (i + 2)T. In order to have all elements of \( \mathbf{z}'(0) \) available simultaneously at \( (i + 2)T \), the kth processor stores the sufficient statistic from kth matched filter for \( T - T_k \) seconds. Recall from (3.13) that to obtain the mth stage estimate of \( \mathbf{b}'(0) \), one needs at most the estimates \( \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1) \). Repeating the argument, the one-stage estimates \( \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1) \) are required to obtain \( \mathbf{b}^{(m)}(m) \). This accounts for a delay of \( (m-1)T \). Further, each stage of multiple-access interference rejection takes a bit duration by assumption.\(^4\) Define decoding delay \( T_{D}(m) \) for the mth stage estimate of a bit to be the time that elapses from the availability of its first-stage estimate to the availability of its mth stage estimate. Accounting for the normalizing delay \( T - T_k \) for the kth user, \( T_{D}(m) = 2(m-1)T \). For instance, at time \( (i + 2)T \) when \( \mathbf{b}^{(1)}(1) \) is observed, the M-stage receiver makes available \( \mathbf{b}^{(2)}(m-1)(m) \) for each \( m = 1, 2, \ldots, M \), as shown in Fig. 3. We therefore have a fixed decoding delay implementation.

Consider the storage requirements of this implementation. The vector of decision statistics \( \mathbf{z}^{(m-1)}(m-1) \) for each \( m = 1, 2, \ldots, M \) require the estimates \( \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1), \mathbf{b}^{(m-1)}(m-1) \) which are obtained at \( iT \), \( (i+1)T \) and at the present time \( (i+2)T \), respectively. It is clear that at any given time, we need to store the current bit estimate and the previous two bit estimates of each user for each \( m = 1, 2, \ldots, M - 1 \) and the storage required is \( 3(M - 1)K \). Moreover, the same storage is required for the multistage detector for the general CDMA system.

Let us now consider the computational requirements. Reconstruction of the MA interference in each stage requires no more than \( 2(K - 1) \) additions/bit. Therefore, for an M-stage detector, \( 2(M - 1)(K - 1) \) additions/bit are needed. In the general CDMA system since each sufficient statistic corresponding to \( \mathbf{z}^{(m)}(0) \) is at most an S-vector, and since a different MA interference has to be subtracted from each component, no more than \( 2S(M - 1)(K - 1) \) additions/symbol are required. In conclusion, we have shown that the implementation of the multistage detector considered here has a fixed decoding delay and a storage complexity/symbol given by \( O(MK) \) and a computational complexity/symbol given as \( O(SMK) \).

Finally, in contrast to the conventional receiver, the multistage receiver, like the optimum receiver, requires a knowledge of the signal energies in addition to the time and phase synchronization with signals of all the users. The problem of estimating signal energies during a training period has been dealt with in [17]. It remains to be seen what performance gains can be achieved with the multistage receiver over the conventional demodulator. This is the topic of the next section.

\(^4\)For convenience, construction and subtraction of multiple-access interference is assumed to require a time duration \( T \) and sign determination is assumed to be instantaneous.
IV. Probability of Error Analysis

The use of a CDMA system for multiple-access communications is motivated in part by its ability to reject disturbances due to multipath, jamming—intentional or otherwise, etc. The decoding operation (despreading in spread-spectrum systems), spreads the spectrum of these external disturbances. Even when the ambient noise is low, the addition of the spreading disturbance will produce the effect of a higher level of ambient noise. Hence, the performance of CDMA receivers is of interest for high as well as low signal-to-noise ratios. Bit-error probability as a function of signal-to-noise ratio is therefore the relevant performance measure.\(^5\)

An exact expression for the probability of error is obtained for the two-stage receiver in a K-user BPSK-CDMA system. Without loss of generality, consider the demodulation of the zeroth bit of the first user. Denoting the bit error probability for the 2-stage receiver as \( P_e(2) \), we have that

\[
P_e(2) = \frac{1}{2} \left[ \mathbb{P}[\epsilon_1^{0}(1) \geq 0 | b_1^{0} = -1] + \mathbb{P}[\epsilon_1^{0}(1) < 0 | b_1^{0} = +1] \right]
\]

where \( \epsilon_1^{0}(1) \) is obtained from (3.12). From now on, we will drop the subscript 1 and the superscript 0 from \( \eta_1 \) and \( b_1 \) to replace them by \( \eta \) and \( b \), respectively. Let \( \beta_1^{0} = [\beta_1^{0}_1, \ldots, \beta_1^{0}_K] \) be the \((K - 1)\)-dimensional column vectors that represent the left and right bits that interfere with the bit \( b_i^{0} \). Each of these vectors for \( i = 0 \) is illustrated in Fig. 4. Notice that \( \beta_1^{0} \) can be written as \( \beta_1^{0} = [\beta_1^{0}_1, \ldots, \beta_1^{0}_K] = \left[\begin{array}{c} \beta_1^{0}_1 \\ \vdots \\ \beta_1^{0}_K \end{array} \right] \).

Further, define the matrix \( \beta_1 = [\beta_1^{0}] \) to represent the bits that directly overlap with \( b \) and \( \beta_2 = [\beta_2^{0}] \) to denote the bits that occur in the decision statistic under consideration but are not and let the error vector \( \beta_1^{0} - \beta_2^{0} \) be defined as \( \delta^{0} \). Analogous to the definition of \( \beta_1 \), define the error vector that occurs in the decision statistic under consideration as \( \delta_1 = \left[\begin{array}{c} \delta_1^{0}_1 \\ \vdots \\ \delta_1^{0}_K \end{array} \right] \).

Similar to the definition of \( \beta_1^{0} \), let us define a \((K - 1)\)-dimensional Gaussian random vector as \( \xi^{0} = \left[\begin{array}{c} \eta^{0}_1 \\ \vdots \\ \eta^{0}_K \end{array} \right] \). Also let \( \xi_1 \) be a Gaussian random variable as \( \xi_1^{0} = (\xi_1^{0}_1, \ldots, \xi_1^{0}_K) \).

It is clear from (3.12) that the decision statistic \( z_1^{0}(1) \) is a function not only of \( \eta \), \( b \), and \( \beta_1 \) but also of \( \beta_2 \) and \( \xi_1 \) because of its dependence on \( \delta_1 \). The decision statistic can be written to show the dependence on these parameters as

\[
z_1^{0}(1) = E_i b + \eta + I(b_i, \xi_1, \delta_1).
\]

The first, second, and the third term will be referred to as the desired signal, additive noise, and residual interference, respectively.

The residual interference is a nonlinear function of the Gaussian random vector \( \xi_1 \). In fact, it is a discrete valued function taking on a specific value for each value of the error vector \( \delta_1 \). The random variables forming \( \xi_1 \) are correlated with \( \eta \) since they are obtained by integrating the product of the corresponding signal and a common white noise process over overlapping intervals. The \((2K - 1)\)-dimensional zero-mean Gaussian random vector \( \xi_1 = [\xi_1^{0}_1, \ldots, \xi_1^{0}_K] \) has a covariance matrix given as

\[
E[\xi_1^{0} \xi_1^{0T}] = \frac{N_0}{2} \begin{bmatrix}
H(0) & H(-1) & Hc(-1) \\
H(1) & H(0) & Hc(0) \\
H(-1) & Hc(0) & Hc(0) \\
\end{bmatrix}
\]

where these blocks are derived from the following decomposition of \( H(i) \) defined in (2.3):

\[
H(i) = \begin{bmatrix}
H_1(i) & H_2(i) \\
H_2(i) & H_1(i) \\
\end{bmatrix}
\]

where \( H_1(i) \) is a \((K - 1) \times 1\) column vector and \( H_2(i) \) is a \(1 \times (K - 1)\) row vector and \( H(i) \) is a \((K - 1) \times (K - 1)\) matrix. Therefore, the additive noise and the residual interference are statistically dependent.

Let us now define the parameters that will be needed in describing the error probability derivation. Define the external values of the sum of the desired signal component and the residual interference for a given \( b \) and \( \beta_1 \) as

\[
L_b(\beta_1) = \min_{\delta_1} \{ -E_i b - I(b, \xi_1, \delta_1) \}
\]

\[
U_b(\beta_1) = \max_{\delta_1} \{ -E_i b - I(b, \xi_1, \delta_1) \}.
\]

Using the fact that an element of the error vector \( \delta_1 \), is such that \( [\delta_1]_i \in \{0, 1\} \), it can be verified that for \( b \in \{+1, -1\} \), we can write

\[
L_b(\beta_1) = -E_i b - 2 \sum_{j=2}^{K} (|b_j| - 1) h_{ij}(1) \]

and

\[
U_b(\beta_1) = -E_i b - 2 \sum_{j=2}^{K} (|b_j| - 1) h_{ij}(0) \]

where \( |x|^+ \) is equal to \( x \) if \( x > 0 \) and is equal to zero otherwise. Similarly, \( |x|^\pm \) is equal to \( x \) if \( x < 0 \) and is equal to zero otherwise.

The general strategy for the evaluation of bit error probability will now be described. Let us first condition on \( b, \beta_1 \), and \( \eta \). The probability of error can be written as

\[
P_e(2) = E_{\eta,b} \left[ E_b(\eta) \left[ P_e(\eta | b, \beta_1, \eta) \right] \right]
\]

where \( E_{\eta,b} \) denotes expectation over the ensemble of independent, uniformly distributed \( b, \beta_1 \in \{-1, 1\}^{2K - 1} \) and similarly, \( E_b \) denotes expectation over the Gaussian random variable \( \eta \). Partitioning the range of \( \eta \) into three intervals defined as \( \eta_1 = \{L_b(\beta_1), U_b(\beta_1)\} \), and \( \eta_2 = \{U_b(\beta_1), \infty\} \), the term in the square bracket in (4.20) can be expressed as

\[
P_e(\eta | b, \beta_1, \eta) = E_b(\eta) P_e(\eta | b, \beta_1, \eta)
\]

\[
+ E_b(\eta_1) P_e(\eta_1 | b, \beta_1, \eta) + E_b(\eta_2) P_e(\eta_2 | b, \beta_1, \eta)
\]

(4.21)
where $L_k(\eta)$ denotes the indicator function of the interval $A_k$. The rest of the derivation deals with the evaluation of the three terms in the above expression.

Let us first begin with the first and the third terms in (4.21) because they are relatively simple to obtain. Recall the definitions of $L_k(\beta_1)$ and $U_k(\beta_1)$ in (4.19). Observe from (4.16), that for a given $\beta_1$, if $b = +1$ and $\eta < L_{-1}(\beta_1)$, then $\text{sgn}(z_k^{(0)}) = 1$. Hence, irrespective of what the error vector $\delta_1$ may be, no error occurs. Conversely, if $b = +1$ and $\eta < L_{-1}(\beta_1)$ or if $b = -1$ and $\eta > U_{-1}(\beta_1)$, then $\text{sgn}(z_k^{(0)}) = -1$. In this case, irrespective of what $\delta_1$ is, an error occurs. Summarizing,

$\Pr[\text{error } | b, \beta_1, \eta] = \begin{cases} 1 & \text{if } b = +1 \text{ and } \eta < L_{-1}(\beta_1) \\ 0 & \text{if } b = -1 \text{ and } \eta < L_{-1}(\beta_1) \end{cases}$, \hspace{1cm} (4.22)

and

$\Pr[\text{error } | b, \beta_1, \eta] = \begin{cases} 0 & \text{if } b = +1 \text{ and } \eta > U_{-1}(\beta_1) \\ 1 & \text{if } b = -1 \text{ and } \eta > U_{-1}(\beta_1) \end{cases}$, \hspace{1cm} (4.23)

Substituting (4.23) and (4.22) into the first and the third terms, respectively, of the expression in (4.21), and then taking the expectation of each term over $\eta$ and then over $\beta_1$ and $b$, it can be shown that

$E_{b, \beta_1}[E_r[I_{A_2}(\eta) \Pr[\text{error } | b, \beta_1, \eta]]] = \frac{1}{2} E_b \left[ Q\left(\sqrt{\frac{2}{N_0 E_1} L_{-1}(\beta_1)}\right) \right], \hspace{1cm} (4.24)$

and

$E_{b, \beta_1}[E_r[I_{A_2}(\eta) \Pr[\text{error } | b, \beta_1, \eta]]] = \frac{1}{2} E_b \left[ Q\left(\sqrt{\frac{2}{N_0 E_1} U_{-1}(\beta_1)}\right) \right]$, \hspace{1cm} (4.25)

where $Q(x)$ is the complementary error function so that $Q(x) = 1/\sqrt{2\pi} \int_x^{\infty} e^{-t^2/2} dt$.

Next, consider the evaluation of the second term in (4.21). For a given $b$ and $\beta_1$, and for a realization of $\eta$ in $A_2$ denoted by $\chi$, the set of error vectors that give rise to an error can be written as

$\Delta_b(\chi, \beta_1) = \{ \delta_1 \in D(\beta_1) \text{ such that } \text{sgn}[x + E_b b + I(\beta_1, \beta_2, \xi_k, \delta_1)] = -b \}$

where $D(\beta_1)$ is the set of all $2^{2K-1}$ admissible error vectors $\delta_1$. Since every $\delta_1 \in \Delta_b(\chi, \beta_1)$ yields a disjoint hyperquadrant in $(z_1^{(0)}, \ldots, z_K^{(0)})$, the probability of this event can be expressed as

$\Pr[\text{error } | b, \beta_1, \eta = \chi] = q_b(\chi, \beta_1) = \sum_{\delta_1 \in \Delta_b(\chi, \beta_1)} \left[ \Pr[\beta_1(1) = \beta_1, \beta_2(1) = \chi, \delta_1] \right]$ \hspace{1cm} (4.26)

or alternatively,

$q_b(\chi, \beta_1) = E_{\delta_1}[\Pr[\delta_1 \in \Delta_b(\chi, \beta_1), \beta_2(1) = \chi, \delta_1] \Pr[\beta_1(1) = \delta_1]], \hspace{1cm} (4.27)$

where each element in $\Delta_b(\chi, \beta_1) = [\beta^{(0)}_{-1}(1) \delta^{(0)}_1(1)]$ is equal to the sign of the corresponding zeroth stage statistic that is Gaussian under the conditioning specified in the above equation. Therefore, the probability inside the expectation in (4.26) is equivalent to the evaluation of a $2^{K-1}$-dimensional normal distribution function. For

the evaluation of this probability, the conditional density of $[\xi^{(-1)}_1, \xi^{(0)}_1]$ given $\eta = \chi$ has to be used, which can be deduced from (4.17) and easily shown to be normal with mean vector $\hat{\beta}_1(1)$ and covariance matrix given as

$E_{\delta_1}[\tilde{E}_b(1) \delta^{(0)}_1(1)]$, \hspace{1cm} (4.28)

For details on the efficient computation of the multivariate normal distribution function see [12]. Alternatively, the evaluation of the probability in the expression for $q_b(\chi, \beta_1)$ in (4.27) can be simplified by expressing it as the sum of multivariate normal distribution functions of dimension less than or equal to $2(K - 1)$ by exploiting the nature of the set $\Delta_b(\chi, \beta_1)$. Significant savings in computation result from a direct evaluation of the probability $q_b(\chi, \beta_1)$.

The conditional density of $[\xi^{(-1)}_k, \xi^{(0)}_k]$ given $\eta = \chi$ depends on $\chi$ and hence does so the probability in (4.27). Substituting (4.27), or equivalently (4.26) in the second term of the expression in (4.21), and taking the expectation over $\eta$, $b$, and $\beta_1$, we have

$E_{b, \beta_1}[E_r[I_{A_2}(\eta) \Pr[\text{error } | b, \beta_1, \eta]]] = E_{b, \beta_1} \left[ \int_{I_{A_2}(\eta)} q_b(x, \beta_1) f_b(x) dx \right]$, \hspace{1cm} (4.29)

where $f_b(x)$ is the marginal probability density function of $\eta$.

Finally, observe from (4.20) and (4.21) that the error probability is simply obtained as the sum of the contributions from the error events corresponding to $\eta$ belonging to the intervals $A_1, A_2$, and $A_3$ from (4.24), (4.25), and (4.29), respectively. Therefore,

$P_r(2) = E_{b, \beta_1} \left[ Q\left(\sqrt{\frac{2}{N_0 E_1} L_{-1}(\beta_1)}\right) \right] + \left[ Q\left(\sqrt{\frac{2}{N_0 E_1} U_{-1}(\beta_1)}\right) \right] + E_{b, \beta_1} \left[ \int_{I_{A_2}(\eta)} q_b(x, \beta_1) f_b(x) dx \right]$, \hspace{1cm} (4.30)

Note, however, that for a given $\beta_1$, we need to compute the probabilities in (4.26) or equivalently in (4.27) for each evaluation of the function $q_b(\chi, \beta_1)$, before it can be integrated in the interval $[L_b(\beta_1), U_b(\beta_1)]$. There are $2^{2^{K-1}}$ such integrals to compute. Needless to say, the evaluation of the exact probability of error is computationally intensive and depends exponentially on the number of users. The intensive computational requirement for analyzing this receiver is partly due to the fact that the various Gaussian noise variables that appear in the decision statistic for the two-stage receiver are all correlated with each other. Furthermore, the decision statistic is a nonlinear function of these variables.

Let us now consider the bit-error probability of the conventional detector in a $K$-user (Gaussian) channel. Again, without loss of generality, consider the demodulation of the zeroth bit of the first user. It is easily shown from (3.15), (3.6) and using the notation from (4.18) that this error probability can be expressed as

$P_r(1) = E_{\delta_1} \left[ \int \left( \frac{2}{N_0 E_1} E[-1] \bar{H}_{\delta_1(1)}(1) \delta^{(0)}_1(1) + \bar{H}_{\delta_1(0)}(0) \delta^{(0)}_1(1) \right) \right]$. \hspace{1cm} (4.31)

The expression in (4.31) will be evaluated in the following section to compare the performance of the conventional detector to the two-stage receiver.
V. Numerical Results

In this section, we present several numerical examples to illustrate the comparison of three demodulators: the conventional detector, the two-stage detector and the matched filter operating in the absence of interfering signals. The first set of examples depict the performances as the signal cross-correlations and their relative signal strengths are varied. Then, we evaluate the performance of these detectors for an important class of BPSK-CDMA systems which employ direct-sequence spread-spectrum (DS-SS) signaling for their multiple-access capability. Several numerical examples for a two-user channel are presented for both low and high bandwidth efficiencies and also the "near-far" problems.

A. Enhanced Bandwidth Efficiency and Near-Far Immunity

For a two-user channel, the performance of any detector would depend on four variables: the normalized signal cross-correlations \( \rho_{12} = (E_r/E_s)^{-1/2} \rho_{12}(0) \) and \( \rho_{12} = (E_r/E_s)^{-1/2} \rho_{12}(1) \), the signal-to-noise ratios of the two users denoted SNR, and SNR, respectively. Figs. 5 and 6 illustrate the logarithm of the error probability of the first user as a function of \( \rho_{12} \) and \( \rho_{21} \) with \( \text{SNR}_1 = 8 \) dB. In Fig. 5, the signal strength of the second user is the same as that of the first user. In Fig. 6, a near-far situation is considered with \( E_2/E_1 = 3 \) dB. That is, the interfering user is about twice as strong as the desired user. Fig. 7 depicts the logarithm of the error probability of the first user as a constant of the correlations \( \rho_{12}, \rho_{21} \) and the energy ratio \( E_2/E_1 \) in dB when \( \rho_{12} \) is set equal to \( \rho_{21} \).

Bandwidth efficiency has a direct bearing on the signal correlations. In general, higher bandwidth efficiencies result in larger correlations. From Figs. 5 and 6 it can be observed that the region of reliable demodulation in the cross-correlation space has been extended considerably by the two-stage detector. Higher bandwidth efficiencies are therefore achieved. Also, notice that the extension is greater when the interfering signal is stronger. This fact suggests that the probability of error comparison would be of interest as a function of the relative signal strengths in addition to the signal cross-correlations. Fig. 7 illustrates the fact that the region of effective detection becomes larger as the interfering signal strength increases. The improvement over the conventional receiver in near-far situations is therefore two-fold since the conventional receiver degrades with the increase of the interfering signal strength. The less dramatic improvement for large correlations suggests the need for employing alternative initial estimates and/or multiple stages. Finally, the degradation of the two-stage detector as compared to the conventional detector for a small enough interfering signal strength points to the need for a higher stage detector. For example, in a three-stage detector a good estimate of the second user's signal is made in the second stage (since the first user's signal strength is relatively higher) which is then used in the third stage to reject the MA interference more reliably than was possible in the second stage.

B. Direct-Sequence Spread-Spectrum Systems

Next, we consider some examples with direct-sequence spread-spectrum signaling. The details of the system description can be found in a number of references (see, for example, [13]). However, the essentials will be repeated here for convenience. For DS-SS signals, the time-limited bandpass signal \( s_k(t) \) in (2.2) can be written as

\[
s_k(t) = \sqrt{2E_r} T^{-1} a_k(t) \cos (\omega_c t + \theta_k)
\]

where \( \omega_c \) is the carrier frequency and \( \theta_k \) is the phase angle. In this format, the direct-sequence spreading waveform \( a_k(t) \) assigned to the \( k \)th user is given as

\[
a_k(t) = \sum_{j=0}^{N-1} a^{(j)} c(t - jT_c); a^{(j)} \in \{+1, -1\}
\]
where for each \( k \), \( \{b'_k \}_{l=1}^{N-1} \) is the code sequence assigned to the \( k \)th transmitter. The chip waveform \( c(t) \) is assumed to be a unit rectangular signal restricted to be zero outside the chip interval \([0, T_c] \) and the number of chips per bit is \( N \). If \( a_kT \) is an integer multiple of \( 2\pi \) and \( > 1 \), the matrices \( H_i(t) \) defined in Section II can be related to the normalized baseband correlation functions as in [17] so that

\[
\begin{align*}
\hat{h}_{10}(0) &= \frac{(E_1E_1)^{1/2}}{N} R_{11}(T_c) \cos(\phi_{11}) \text{ if } l \leq k \\
\hat{h}_{10}(1) &= 0 \text{ if } l \leq k \\
\hat{h}_{10}(l) &= \frac{(E_1E_1)^{1/2}}{N} R_{11}(T + t_c - \tau_i) \cos(\phi_{11}) \text{ if } l > k
\end{align*}
\]  

(5.33a) (5.33b)

where \( R_{11}(\tau) \triangleq T_c^{-1} \int_0^{T_c} a_k(t) a_k(t+\tau)dt \) for \( 0 \leq \tau \leq T_c \) is the partial continuous-time correlation function [15]. The relative phase delay is defined as \( \phi_{11} = \phi_{11} \), \( \phi_{21} = \phi_{21} \), \( \phi_{31} = \phi_{31} \), and \( \phi \triangleq (\phi_{11}, \phi_{21}, \cdots, \phi_{31})^T \). The data bits \( \{b'_l \}_{l=1}^{N-1} \) are independent and equally likely to be +1 or -1.

The probability of bit-error of the first user for the one-stage receiver given in (4.31) is now denoted as \( P_e(1)(\tau, \phi) \) where \( \tau = (\tau_1, \tau_2, \cdots, \tau_k)^T \) and \( \phi \triangleq (\phi_1, \phi_2, \cdots, \phi_k)^T \), to show its explicit dependence on the time and phase delays of all the users in the system. It can be shown that for each \( \tau \),

\[
P_e(1)(\tau, \phi) \leq P_e(1)(\tau, \phi) \big|_{\phi=0},
\]

using the convexity of the \( Q \)-function and assuming the eye-open condition [8]. That is, the base-band case is the worst case for each set of relative time delays. Further, under the same condition, we can show that the error probability, averaged over independent and uniformly distributed time and phase delays with \( \tau \in [0, T_c]^k \) and \( \phi \in [0, 2\pi]^k \), is upper bounded by its average for the base-band case on a discrete grid of time delays corresponding to the chip boundaries, i.e.,

\[
\frac{1}{(2\pi)^k} \int_{\tau_1}^{T_c} P_e(1)(\tau, \phi) d\tau d\phi \\
\leq \frac{1}{N^k-1} \sum_{\tau \in [0, T_c]^k} P_e(1)(\tau, \phi) \big|_{\phi=0}
\]

(5.34)

Moreover, the worst-case error probability occurs on the worst-case grid of time delays. Therefore, we have

\[
\max_{\tau, \phi} P_e(1)(\tau, \phi) = \max_{\tau \in [0, T_c]^k} P_e(1)(\tau, \phi) \big|_{\phi=0}
\]

(5.35)

These and similar results for the upper bound of the maximum-likelihood sequence detector were obtained in [17]. Therefore, in order to conduct a meaningful comparison between the conventional, the two-stage and the optimum receiver, we will compute

\[
P_e(2) \triangleq \frac{1}{N^k-1} \sum_{\tau \in [0, T_c]^k} P_e(2)(\tau, \phi) \\
\]

(5.36)

\[
P_e(2) \triangleq \max_{\tau \in [0, T_c]^k} P_e(2)(\tau, \phi)
\]

(5.37)

for the two-stage receiver for comparison to the corresponding probabilities of the conventional and the optimum receivers.

Two sets of examples are considered corresponding to both low and high bandwidth efficiencies (defined by the \( K/N \) ratio). For each set, the near-far situation is also illustrated. These examples were considered for presenting numerical results in [18] and therefore comparison to the performance of the optimum receiver can be made.

In the first set of examples in Fig. 8, we consider two spread-spectrum signals which are maximal-length signature sequences of length 31 generated to maximize a signal-to-multipath-access-interference functional [6]. Average error probability of the conventional detector for these signature waveforms in the equal-energy situation has been extensively studied [7], [14]. Figs. 8(a)-(c) depict the worst case (1- and 2-STAGE U.B. in legend) the upper bound on the average error probability (1- and 2-STAGE A.V. In legend) for the conventional and the two-stage receiver in comparison to the single user error probability for \( E_2/E_1 \) (in dB) equal to -1 dB, 0 dB, and +3 dB, respectively, corresponding to the second user being approximately half, equal and twice as strong as the first user. Notice the improvement in the performance of the two-stage receiver as the second users' signal strength increases. This behavior was also noted in [3] where it was shown that the interference channel capacity coincides with that of a single-user channel and in [19] where it was shown that the asymptotic efficiency of the optimum detector is equal to one for a sufficiently high interference. The simple explanation for this is that, if the second user has a high energy, it is estimated better by the first stage and hence is rejected more successfully.

The second set of examples [see Fig. 9(a)-(d)] correspond to high bandwidth efficiency situations. The code waveforms of the two users are depicted in Fig. 9(a). Fig. 9(a)-(d) depict the worst-case error probabilities for the relative signal strengths shown in each of the graphs. Here, the worst case error probability and the upper bound on the average error probability are the same for the one-stage and also for the two-stage receivers. Notice a similar trend again in this high bandwidth efficiency case. Fig. 9(d) illustrates the near-invariance of the error probability of the two-stage receiver as a function of the signal strength of the second user. The near-far problem is therefore alleviated. The conventional receiver on the other hand, degrades due to a strong interfering signal.
VI. Conclusions

Optimum multiuser demodulation for coherent communications in CDMA channels is inherently a difficult problem requiring intensive computation and a variable decoding delay that is unacceptable in most practical applications. On the other hand, the conventional multiuser demodulator suffers severe degradation in the useful ranges of bandwidth efficiencies and in near-far situations. There is hence a need for suboptimum detectors which are robust to near-far effects with a reasonable computational complexity to ensure their practical implementation. In this paper, a multi-stage multiuser detector is proposed which is based on a successive multiple-access interference rejection scheme. This algorithm has a computational complexity that is linear in the number of users. Also, an efficient implementation of the proposed detector is demonstrated.

The probability of error of the two-stage receiver was obtained. Its computation is complicated because the decision statistic depends in a nonlinear form on some normal random variables which are dependent. From the evaluation of the probability of error, it was shown that the region of reliable demodulation in the cross-correlation space was extended considerably even by the two-stage detector. This extension was larger as the interfering signal strength increased. It is this feature that makes the multistage detector eminently suitable for demodulating relatively weak signals in the presence of strong interfering signals. The near-far problem was therefore alleviated. However, the improvement for large signal cross-correlation was less significant, thereby motivating the need for employing better initial estimates and/or higher-stage detectors for this region. Similar conclusions were obtained from two sets of direct-sequence spread-spectrum examples corresponding to low and high bandwidth efficiencies.

In conclusion, the proposed multiuser demodulators would prove to be valuable in situations where there is need for partial/full centralized demodulation as well as situations where there are partial/no security restrictions within the network. For these situations, some or all of the code waveforms are available at the receiver. The corresponding time and phase synchronization capabilities are required (centralized demodulation) or can be made available. The modification of the multiuser receiver to take into account only a subset of all the users can be made easily.
APPENDIX

PROCESSING TIME INTERVAL FOR M-STAGE DETECTORS

In this Appendix, the effective time interval over which the M-stage multistage detector processes the received signal in demodulating any symbol is obtained. Consider the ith symbol of the 4th user, the M-stage receiver using the conventional detector’s decision as the first stage, effectively processes the received signal waveform in the interval given as

\[-(M - 1)T + \tau_{g1}T + \tau_{r4(k+M-1)} + iT\]

where \(\tau_{g4} = (k + M - 1) - \sigma_k(k + M - 1), K\) \(\sigma_k = (k - M + 1) - \sigma_k(k - M + 1),\) and \(\sigma_k(j) = 1 + (j - 1) \mod K.\) Excluding the symbol under consideration, in the general case there are 2M(K - 1) information symbols associated with this interval.

We will arrive at this processing interval by considering the following argument on the processing time of each stage. The 1st-stage or the conventional decision for \(b_{4k}^1\) is based on \(z_{4k}^1(0)\) which is obtained from the received signal in the interval \(\tau_{g1}T, \tau_{r4}T + (i + 1)T.\)

The 2nd-stage decision is based on \(z_{4k}^1(1)\) which is obtained by rejecting the MA interference in \(z_{4k}^1(0)\) based on the 1st-stage estimates of bits that overlap with \(b_{4k}^1.\) Since it is assumed that \(0 < \tau_1 < \tau_2 < \cdots < \tau_K < T,\) the left-most bit that overlaps with \(b_{4k}^1\) denoted as \(L(b_{4k}^1)\) is \(z_{4k}^1(l)\) if \(k + 1 < K\) or \(b_{4k}^1\) if \(k = K\) (lower end of the processing interval at this stage is \(\tau_r + (l - 1)T\) or \(\tau_r + iT,\) respectively). Again, the 3rd-stage decision statistic \(z_{4k}^2(2)\) is obtained by rejecting the MA interference based on 2nd-stage decisions of symbols that overlap with \(b_{4k}^1.\) The 2nd-stage decision on \(L(b_{4k}^2)\) requires its 1st-stage decision on \(L(L(b_{4k}^1))) = I^2(b_{4k}^1)\) which in turn is \(b_{i+2}^{(2)}\) if \(k + 2 < K\) or \(b_{i+1}^{(2)}\) if \(k + 1 = K\) or \(b_{i+1}^{(2)}\) if \(k = K.\) In general, the Mth-stage decision statistic \(z_{4k}^m(M - 1)\) requires \(L^{M-1}(b_{4k}^1)\) in the form of the 1st-stage decision. Following the argument above, it can be easily verified that the left-most bit that occurs in the decision statistic \(z_{4k}^m(m)\) is \(R^m(b_{4k}^1)\) and can be verified to be \(b_{4k+1}^{(m+1)}\) where \(\sigma_k\) is defined in the statement of the remark. Similarly, the right-most bit that appears in \(z_{4k}^m(m)\) is denoted as \(R^m(b_{4k}^1)\) and can be verified to be \(b_{4k+K-m}^{(m+1)}\) where \(\sigma_k\) is defined in the statement of the remark. Therefore, to obtain the Mth stage estimate of \(b_{4k}^1,\) the first stage estimate of the \((K - 1)\) left-most bits and the \((K - 1)\) right-most bits that occur in the decision statistic \(z_{4k}^m(M - 1)\) are needed. Since the first-stage estimates are assumed to be the conventional one-shot decisions, the time interval over which \(\tau_1\) is given as the left time epoch of \(L^{M-1}(b_{4k}^1)\) and the right time epoch of \(R^{M-1}(b_{4k}^1)\).

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Fig. 9. A comparison between the worst case and upper bound of the average error probability of a two-user direct-sequence spread-spectrum system with \(N = 3\) for the conventional receiver and the two-stage receiver and the single-user bit-error probability. (a) \(E_1/E_1 = -3\ dB,\) (b) \(E_1/E_1 = 0\ dB,\) (c) \(E_1/E_1 = 3\ dB.\)
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