

# Capacity Improvement with Base-Station Antenna Arrays in Cellular CDMA

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**Abstract**—In this paper, the use of antenna array at base-station for cellular CDMA is studied. We present a performance analysis for a multicell CDMA network with an antenna array at the base-station for use in both base-station to mobile (downlink) and mobile to base-station (uplink) links. We model the effects of path loss, Rayleigh fading, log-normal shadowing, multiple access interference, and thermal noise, and show that by using an antenna array at the base-station, both in receive and transmit, we can increase system capacity several fold. Simulation results are presented to support our claims.

## I. INTRODUCTION

THE increasing demand for mobile communication services without a corresponding increase in RF spectrum allocation motivates the need for new techniques to improve spectrum utilization. One approach for increased spectrum efficiency in digital cellular is the use of spread spectrum code-division multiple-access (CDMA) technology [1], [2]. Despite the high capacity offered by CDMA technology, the expected demand is likely to outstrip the projected capacity with the introduction of Personal Communication Networks (PCN). One approach that shows real promise for substantial capacity enhancement is the use of spatial processing with cell site antenna array [3]–[11]. By using spatial processing at the cell site, we can estimate the array response vector and use optimum directional receive and transmit beams to improve system performance and increase capacity. Such improved antenna processing can be incorporated into the proposed CDMA transmission standards. The increase in system capacity by using antenna arrays in CDMA comes from reducing the amount of co-channel interference from other users within its own cell and neighboring cells. This reduced interference transforms to an increase in capacity. The currently proposed IS-95 CDMA standard already incorporates a degree of spatial processing through the use of simple sectorized antennas at the cell site. It employs three receive and transmit beams of width  $120^\circ$  each to cover the azimuth. Sectoring nearly triples system capacity in CDMA. While it might appear that even nar-

rower sectors might yield further capacity gains, simple planar wavefront assumptions used in sectoring are not valid for narrow beams that employ large apertures. Simple sectoring, therefore, suffers from significant losses and motivates the need for "smart antennas" that adapt to a dynamic spatial channel seen by the cell site antenna array.

In this paper, we study the capacity improvement of multicell CDMA cellular system with base-station antenna array for both the downlink and the uplink. As in the proposed IS-95 CDMA standard, we assume that the uplink and the downlink occupy different frequency bands. We adopt the Rayleigh fading and log-normal shadowing model in [12] to model signal level. In this model, the fast fading around the local mean has a Rayleigh distribution. Due to shadowing, the local mean fluctuates around the area mean with a log-normal distribution and standard deviation  $\sigma_s$ , which varies between 6 to 12 dB, depending on the degree of shadowing. We also assume that the received signal power falls off with distance according to a fourth power law. That is, the path loss between the user and the cell site is proportional to  $r^{-4}$  where  $r$  is the distance between user and cell site.

In the next section, we analyze system capacity in uplink, where each signal propagates through a distinct path and arrives at the base station with independent fading. In Section III, we also analyze system capacity in downlink where all signals received at the mobile from the same base station undergo the same fading. Next, in Section IV we present simulation results. Finally, Section V contains our concluding remarks.

## II. MOBILE-TO-BASE LINK

We assume that the cell site alone uses a multielement antenna array to receive and transmit signals from and to the mobile. No antenna arrays are considered for the mobile due to practical difficulties in implementing such a concept. Consider a scenario where there are  $N$  users randomly distributed around each cell site at varying ranges. We assume that the receiver is code locked onto every user but does not know the direction of-arrival (DOA) of these users. Each user transmits a PN code modulated bit stream with a spreading factor (processing gain) of  $L$ . Let  $P$  be the received signal power at the cell site, let the system noise power (excluding interference from other in-

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band users) be  $\sigma^2$  and, finally, let  $M$  be the number of antenna elements. Assuming *perfect instantaneous* power control, the interference from a mobile within a given mobile's cell will arrive at cell site with same power  $P$ . Since mobiles in other cells are power controlled by their cell sites, the interference power from such mobiles, when active, at the desired user's cell site is given by [1]

$$I_{ik} = P \left( \frac{r_{ik}^{(k)}}{r_{ik}^{(o)}} \right)^4 \frac{\|\alpha_{ik}^{(o)}\|^2}{\|\alpha_{ik}^{(k)}\|^2} = P \cdot \beta_{ik}^2 \quad (1)$$

where  $r_{ik}^{(k)}$  is the distance from the  $i_k$ -th user in the  $k$ -th cell to its cell site,  $\alpha_{ik}^{(k)}$  is a zero mean complex Gaussian random variable that represents the corresponding amplitude fade along that path and combines both the Rayleigh fading and log-normal shadowing effects (i.e.,  $\|\alpha_{ik}^{(k)}\|$  has a Rayleigh distribution whose mean square value  $E\{\|\alpha_{ik}^{(k)}\|^2\}$  is log-normal; i.e.,  $10 \log_{10} E\{\|\alpha_{ik}^{(k)}\|^2\}$  is normally distributed with zero mean and variance  $\sigma_s^2$ ),  $r_{ik}^{(o)}$  is the distance between the same  $i_k$ -th mobile in the  $k$ -th cell and the desired user's cell site (i.e., cell site  $o$ ), and finally  $\alpha_{ik}^{(o)}$  is the corresponding amplitude fade. Note that in [1], only the effects of the log-normal shadowing is considered. Note also that since the mobile will be controlled by the cell site that has minimum attenuation  $\beta_{ik} \leq 1$  [1].

Fig. 1 shows a desired signal and interference signals from mobiles within cells and outer cells for both omnidirectional beams and directional beams. Clearly, directional beams reduce the interference power and boost the signal to interference-plus-noise ratio. To be able to form such beams, we need to estimate the array response vector, or the spatial signature, of the desired user mobile. Using this estimate of the array response vector, we can form a beam towards each mobile.

Assuming a narrowband signal model, the  $M \times 1$  output of an array of  $M$  sensors at the cell site can be written as

$$\begin{aligned} \mathbf{x}(t) = & \sum_{i_o=1}^N \psi_{i_o} \sqrt{P} b_{i_o} \left( \left\lfloor \frac{t - \tau_{i_o}}{T} \right\rfloor \right) c_{i_o}(t - \tau_{i_o}) \mathbf{a}_{i_o} \\ & + \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} \sqrt{P} \beta_{i_k} b_{i_k} \left( \left\lfloor \frac{t - \tau_{i_k}}{T} \right\rfloor \right) \\ & \cdot c_{i_k}(t - \tau_{i_k}) \mathbf{a}_{i_k} + \mathbf{n}(t) \end{aligned} \quad (2)$$

where  $K$  is the number of interfering cells,  $\mathbf{a}_{i_k}$  is the  $M \times 1$  array response vector for signal arriving from the  $i_k$ -th mobile in the  $k$ -th cell and we assume that  $\mathbf{a}_{i_k}^* \mathbf{a}_{i_k} = 1$ ,  $c_{i_k}(t)$  is the code used by that user,  $b_{i_k}(\cdot)$  is the bit of duration  $T$ ,  $\tau_{i_k}$  is the propagation delay,  $\psi_{i_k}$  is a Bernoulli variable with probability of success  $v$  that models the *voice activity* of the same user (i.e., a user will be talking with probability  $v$ ), and  $\mathbf{n}$  is the thermal noise vector with zero-mean and covariance

$$E\{\mathbf{n}(t)\mathbf{n}^*(\tau)\} = \frac{\sigma^2}{M} \mathbf{I} \quad t = \tau \quad (3)$$

$$= \mathbf{0} \quad t \neq \tau. \quad (4)$$

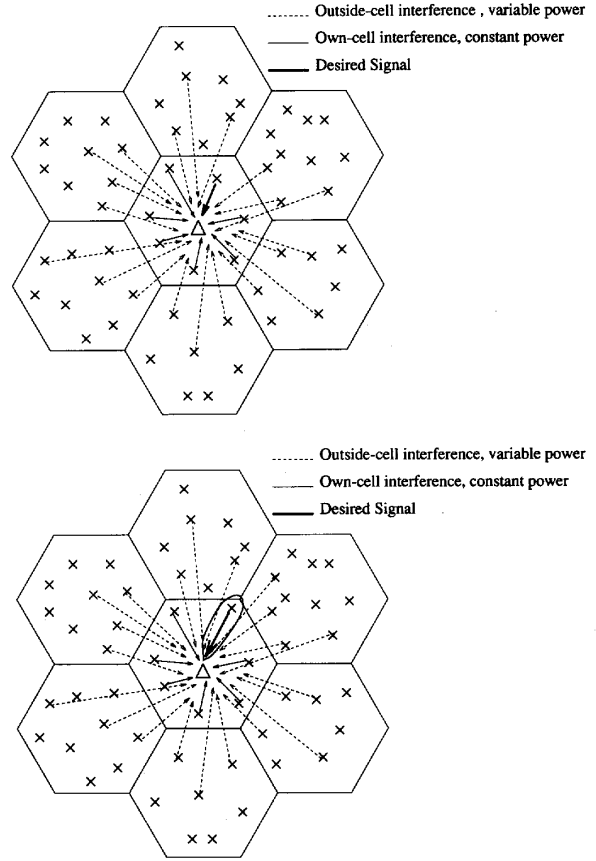


Fig. 1. Interference in uplink with and without beamforming.

Equation (3) implies that the noise is both *temporally* and *spatially* white. For the desired user, let  $\mathbf{a}_o$ ,  $\tau_o$ ,  $c_o$ , and  $b_o(\cdot)$  be the array response vector, the time delay, the used code, and the transmitted bits, which are assumed to be i.i.d. binary random variables taking values  $\pm 1$  with equal probability, respectively. The antenna outputs are correlated with the desired user's code  $c_o$  to yield one sample vector per bit. Without loss of generality, assume that  $\tau_o = 0$ . The post-correlation signal vector for the desired user's  $l$ -th bit is given by

$$\begin{aligned} \mathbf{z}_o(l) &= \int_{t_1}^{t_2} \mathbf{x}(t) c_o(t) dt \\ &= s_o(l) \mathbf{a}_o + \sum_{i_o=2}^N \psi_{i_o} I_{i_o}(l) \mathbf{a}_{i_o} \\ &\quad + \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} I_{i_k}(l) \mathbf{a}_{i_k} + \mathbf{n}_T(t) \end{aligned} \quad (5)$$

where  $t_1 = (l-1)T$ ,  $t_2 = lT$ , and

$$s_o(l) = \int_{t_1}^{t_2} \sqrt{P} b_o \left( \left\lfloor \frac{t}{T} \right\rfloor \right) c_o(t) c_o(t) dt, \quad (6)$$

$$I_{i_o}(l) = \int_{t_1}^{t_2} \sqrt{P} b_{i_o} \left( \left\lfloor \frac{t - \tau_{i_o}}{T} \right\rfloor \right) c_{i_o}(t - \tau_{i_o}) c_o(t) dt, \quad (7)$$

$$I_{i_k}(l) = \int_{t_1}^{t_2} \sqrt{P} \beta_{i_k} b_{i_k} \left( \left\lfloor \frac{t - \tau_{i_k}}{T} \right\rfloor \right) c_{i_k}(t - \tau_{i_k}) c_o(t) dt, \quad (8)$$

$$n_T(l) = \int_{t_1}^{t_2} c_o(t) n(t) dt. \quad (9)$$

In order to combine the array outputs to estimate the signal from the desired mobile, we need to determine the array response vector for the wavefront arriving from this user. In general, in CDMA systems the number of users will far exceed the number of antennas. Therefore, subspace methods of direction-of-arrival estimation (e.g., MUSIC [13] and ESPRIT [14]) are not applicable. In [3], we showed that the array response vector of the desired mobile  $\mathbf{a}_o$  can be estimated from the pre-correlation and post-correlation array covariances  $\mathbf{R}_{xx}$  and  $\mathbf{R}_{z_o z_o}$ , where

$$\mathbf{R}_{xx} = \mathcal{E} \{ \mathbf{x} \mathbf{x}^* \}$$

$$\mathbf{R}_{z_o z_o} = \mathcal{E} \{ \mathbf{z}_o \mathbf{z}_o^* \}$$

as the generalized principal eigenvector of the matrix pair  $(\mathbf{R}_{xx}, \mathbf{R}_{z_o z_o})$ . Using this estimate of  $\mathbf{a}_o$ , the post-correlation antenna outputs are combined via beamforming to estimate the signal from the desired user. The decision variable, which is the output of the beamformer, is then given by

$$\begin{aligned} d_o(l) &= \mathbf{a}_o^* \mathbf{z}_o(l) \\ &= s_o(l) + n_1(l) + n_2(l) + n_T(l) \\ &= L \sqrt{P} b_o(l) + \sum_{i_o=2}^N \psi_{i_o} I_{i_o} \mathbf{a}_o^* \mathbf{a}_{i_o} \\ &\quad + \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} I_{i_k} \mathbf{a}_o^* \mathbf{a}_{i_k} + \mathbf{a}_o^* \mathbf{n}_T(l). \end{aligned} \quad (10)$$

The first term  $s_o(l)$  is due to signals from the desired user, the second term  $n_1$  is due to interference from users within its own cell, the third term  $n_2$  is due to interference from users outside the cell; both are zero mean, and  $n_T$  is due to the additive thermal noise, which is normal with zero mean and variance equal to  $\text{Var} \{ n_T \} = L \sigma^2 / M$ . Additionally, we assume that each user's code consists of a sequence of  $L$  i.i.d. binary random variables taking values  $\pm 1$  with equal probability. As noted in [15], with asynchronous transmission, random-sequence codes give approximately the same analytical results for nonrandomly chosen codes. Under this assumption and using the results in [15], we can show that the variances of  $n_1$  and  $n_2$  are given by

$$\text{Var} \{ n_1 \} = LP \sum_{i_o=2}^N \psi_{i_o} \|\mathbf{a}_o^* \mathbf{a}_{i_o}\|^2, \quad (11)$$

$$\text{Var} \{ n_2 \} = LP \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} \beta_{i_k}^2 \|\mathbf{a}_o^* \mathbf{a}_{i_k}\|^2. \quad (12)$$

These variances are themselves random variables that depend on the voice activity of the users, their array response vectors, and fading and shadowing effects. The faded energy-per-bit to interference-plus-noise densities ratio can be written as

$$\frac{E_b}{N_o + I_o} = \frac{L}{\frac{\sigma^2}{MP} + I_1 + I_2} \quad (13)$$

where  $I_1$  and  $I_2$  are the interference-to-signal power ratios due to own cell and outer cell users respectively, and are given by

$$I_1 = \sum_{i_o=2}^N \psi_{i_o} \|\mathbf{a}_o^* \mathbf{a}_{i_o}\|^2, \quad (14)$$

$$I_2 = \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} \beta_{i_k}^2 \|\mathbf{a}_o^* \mathbf{a}_{i_k}\|^2. \quad (15)$$

The probability of outage is defined as the probability of the bit error rate exceeding a certain threshold  $P_o$  required for acceptable performance. As noted in [1], with efficient modems and powerful convolutional codes, adequate performance ( $\text{BER} < 10^{-3}$ ) is achieved with  $E_b / (N_o + I_o) < 7$  dB. Let  $S$  be the  $E_b / (N_o + I_o)$  value required to achieve the level of performance, then the outage probability is

$$\begin{aligned} P_{out} &= \Pr(\text{BER} > P_o) = \Pr \left( \frac{E_b}{N_o + I_o} < S \right) \\ &= \Pr \left( I_1 + I_2 > \frac{L}{S} - \frac{\sigma^2}{MP} \right). \end{aligned} \quad (16)$$

This expression gives the outage probability as a function of the random variables  $I_1$  and  $I_2$ . The distribution of the random variables  $I_1$  and  $I_2$  depends on the number of active users, their relative distances, their array response vectors, array parameters, and fading and shadowing effects. The capacity of the system in terms of maximum cell loading can be found by finding the maximum  $N$  such that for the required BER,  $P_{out}$  will not exceed the present threshold. To obtain  $P_{out}$  as a function of  $N$ , we need to specify the array (i.e., the number of sensors, spacing between them, and their arrangement) to be able to find the distribution of  $\|\mathbf{a}_o^* \mathbf{a}_{i_k}\|^2$ , and hence the distribution of  $I_1$  and  $I_2$ .

To simplify the evaluation, we use the following first order approximation. As we pointed out earlier, the effect of forming a beam towards the desired user is to reduce the effective number of interferers to those mobiles that fall within the beam formed towards the desired mobile. Since the number of those interferers is random, we approximate this effect by replacing the  $\|\mathbf{a}_o^* \mathbf{a}_{i_k}\|^2$  term in  $I_1$  and  $I_2$  by a Bernoulli random variable  $\chi_{i_k}$  that has a probability of success  $B/2\pi$  where  $B$  is the effective bandwidth

and is equal to  $\mathcal{E}\{\|\mathbf{a}_o^* \mathbf{a}_{ik}\|^2\}$ . This random variable represents the *interference activity* of the users, i.e., a mobile will cause interference to the desired mobile if it falls within its beam. In this case, we can write  $I_1$  and  $I_2$  as

$$I_1 = \sum_{i_o=2}^N \psi_{i_o} \chi_{i_o} = \sum_{i_o=2}^N \phi_{i_o} \quad (17)$$

$$I_2 = \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} \chi_{i_k} \beta_{i_k}^2 = \sum_{k=1}^K \sum_{i_k=1}^N \phi_{i_k} \beta_{i_k}^2 \quad (18)$$

where  $\phi_{i_k} = \psi_{i_k} \chi_{i_k}$  is a Bernoulli random variable with probability of success  $\bar{v} = vB/2\pi$ . The distribution of  $f = \|\alpha_{i_k}^{(o)}\|/\|\alpha_{i_k}^{(k)}\|$  is given by [12]

$$\Pr(f < r) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-u^2)}{1 + r^{-2} 10^{-2\sigma_s u/10}} du. \quad (19)$$

For a large number of users, the random variable  $I_2$  (interference due to  $K \cdot N$  users) can be approximated by a Gaussian random variable with mean  $\mu_i N$  and variance  $\sigma_i^2 N$  that depend on  $\bar{v}$ , the degree of shadowing  $\sigma_s$ , and the number of interfering cells  $K$ . We have evaluated the mean and variance of  $I_2$  using Monte Carlo integration considering only the first two tiers of interfering cells (i.e.,  $K = 18$ ) and these were found to be given by

$$\mu_i = 0.523\bar{v}, \quad (20)$$

$$\sigma_i^2 = 0.463\bar{v} - 0.274\bar{v}^2. \quad (21)$$

Also, the random variable  $I_1$  has a binomial distribution with parameters  $(N-1, \bar{v})$ . Let  $L/S - \sigma^2/PM = \delta$ . Since  $I_1$ ,  $I_2$ , and all  $\phi_{i_k}$  are independent, we can use the results in [1] to show that

$$P_{out} = \sum_{k=0}^{N-1} \binom{N-1}{k} \bar{v}^k (1-\bar{v})^{N-1-k} \cdot Q\left(\frac{\delta - k - \mu_i N}{\sqrt{\sigma_i^2 N}}\right) \quad (22)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy.$$

This equation gives the outage probability as a function of the number of mobiles per cell that can be supported. Note that this reduces to the result in [1] when no antenna arrays are used at the cell site. The results of evaluating (22) as a function of cell loading and beamwidth are shown in Fig. 3. Also, simulations to evaluate the accuracy of the above approximation are shown in Fig. 5 and discussed in Section IV.

### III. BASE-TO-MOBILE LINK

Consider now the base-to-mobile link. We assume a similar scenario as in the uplink. With antenna array at the cell site, the cell site must also beamform on the downlink in order to effectively increase the system ca-

capacity. To be able to form such beams, the cell site needs to have an estimate of the transmit array response vector to each mobile. However, in the current standard, frequencies for the uplink and downlink differ by 45 MHz. In this case, the receive and transmit response vectors can be significantly different [16], [17]. Hence, reciprocity between uplink and downlink does not hold and the beamformer weights used for reception cannot be used for transmission. A method of performing transmission beamforming is the feedback method [18], [19], where training signals, or tones, are periodically transmitted from the cell site to all mobiles on the downlink. From the received signal information that the mobiles feedback to the cell site on uplink, it is possible to estimate the downlink spatial channel, and thus estimate the transmit array response vector.

All signals received at the mobile from the same base station will have propagated over the same path, hence they will experience the same fading and path loss. Therefore, we assume that cell site transmits the same power to all mobiles controlled by that cell site. With this assumption, the power of each signal arriving at the desired mobile from the  $k$ -th cell site is given by

$$P_k = P \frac{\|\alpha_k^{(o)}\|^2}{(r_k^{(o)})^4} = P \cdot \beta_k^2 \quad (23)$$

where  $\alpha_k^{(o)}$  represents the fading and shadowing experience by all signals arriving at the desired mobile from the  $k$ -th cell site, and  $r_k^{(o)}$  is the distance between the desired mobile from its cell site. As in [1], we assume that the power received by the mobile from its cell site is the largest among all other signals from other cell sites (otherwise the mobile would switch to the cell site whose received power is maximum). That is, we assume that

$$\beta_o < \beta_k \quad k = 1, \dots, K. \quad (24)$$

Fig. 2 shows desired signal and interference powers seen by the desired mobile for both omni- and directional beams.

Assuming  $N$  users per cell randomly distributed around each cell site at varying ranges, we can write the received signal at the mobile of interest as

$$\begin{aligned} x_o(t) = & \sum_{i_o=1}^N \psi_{i_o} \sqrt{P} \beta_o b_{i_o} \left( \left\lfloor \frac{t - \tau_{i_o}}{T} \right\rfloor \right) c_{i_o}(t - \tau_{i_o}) \mathbf{a}_{i_o}^* \mathbf{a}_o^{(o)} \\ & + \sum_{k=1}^K \sum_{i_k=1}^K \sigma_{i_k} \sqrt{P} \beta_k b_{i_k} \left( \left\lfloor \frac{t - \tau_{i_k}}{T} \right\rfloor \right) \\ & \cdot c_{i_k}(t - \tau_{i_k}) \mathbf{a}_{i_k}^* \mathbf{a}_o^{(k)} + n(t), \end{aligned} \quad (25)$$

where  $n(t)$  is the background noise received by the mobile, and  $\mathbf{a}_o^{(k)}$  is the transmit array response vector of the desired mobile as seen by the  $k$ -th cell site. All other notations remain the same as in the previous section. The mobile correlates the received signal by its code to yield

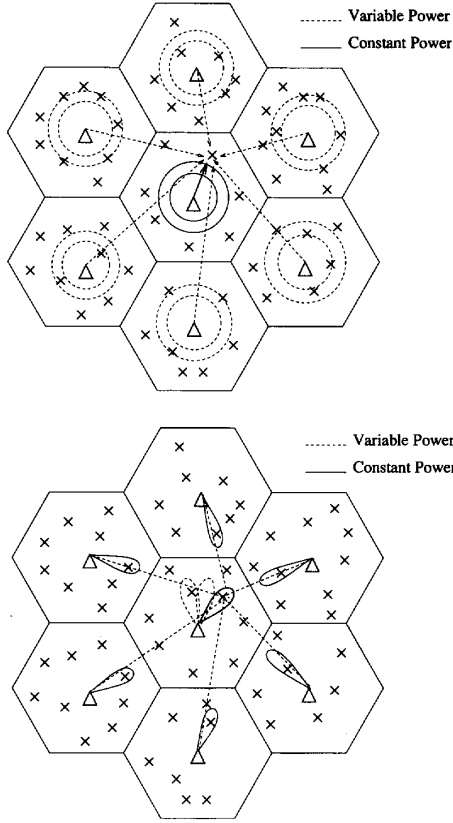


Fig. 2. Interference in downlink with and without beamforming.

the decision variable

$$\begin{aligned}
 d_o &= s_o(l) + n_1(l) + n_2(l) + n_T(l) \\
 &= L\beta_o\sqrt{P}b_o(l) + \sum_{i_o=2}^N \psi_{i_o} I_{i_o} \mathbf{a}_{i_o}^* \mathbf{a}_o^{(o)} \\
 &\quad + \sum_{k=1}^K \sum_{i_k=1}^N \sigma_{i_k} I_{i_k} \mathbf{a}_{i_k}^* \mathbf{a}_o^{(k)} + n_T(l), \quad (26)
 \end{aligned}$$

where  $I_{i_k}$  is defined as before but with  $\beta_k$  instead of  $\beta_{i_k}$  and

$$n_T(t) = \int_{t_1}^{t_2} c_o(t)n(t) dt. \quad (27)$$

As in the uplink, the first term  $s_o$  is due to the desired signal from the cell site to the desired user, the second term  $n_1$  is due to interference from the same cell site into the desired user, which is zero mean, the third term  $n_2$  is due to interference from other cell sites into the desired user, and  $n_T$  is due to the additive thermal noise and it is normal with zero mean and variance equal to  $\text{Var}\{n_T\} = L\sigma^2$ .

In the proposed CDMA standard IS-95, orthogonal codes (Walsh codes) are used on the downlink for all users within a cell, i.e., in the ideal case (no multipath) there is no cross-correlation between those signals and the in-

terference due to signals from its own cell site  $n_1$  is zero. However, we assume here that there will be cross-correlation between those signals, which represents a worst case. Hence, similar to the uplink case, we can show that the variance of  $n_1$  and  $n_2$  is given by

$$\text{Var}\{n_1\} = LP_o \sum_{i_o=2}^N \psi_{i_o} \|\mathbf{a}_{i_o}^* \mathbf{a}_o^{(o)}\|^2 \quad (28)$$

$$\text{Var}\{n_2\} = L \sum_{k=1}^K P_k \sum_{i_k=1}^N \psi_{i_k} \|\mathbf{a}_{i_k}^* \mathbf{a}_o^{(k)}\|^2, \quad (29)$$

and the energy-per-bit to interference-plus-noise densities ratio can be written as

$$\frac{E_b}{N_o + I_o} = \frac{L}{\frac{\sigma^2}{P_o} + G_1 + G_2}, \quad (30)$$

where  $G_1$  and  $G_2$  are the interference-to-signal power ratios due to their own cell and outer cell signals, respectively, and are given by

$$G_1 = \sum_{i_o=2}^N \psi_{i_o} \|\mathbf{a}_{i_o}^* \mathbf{a}_o^{(o)}\|^2 \quad (31)$$

$$G_2 = \sum_{k=1}^K \sum_{i_k=1}^N \psi_{i_k} \frac{P_k}{P_o} \|\mathbf{a}_{i_k}^* \mathbf{a}_o^{(k)}\|^2. \quad (32)$$

The corresponding outage probability is then given by

$$P_{out} = \Pr(\text{BER} > P_o) = \Pr\left(\frac{\sigma^2}{P_o} + G_1 + G_2 > \frac{L}{S}\right). \quad (33)$$

Unlike the uplink case, the distribution of  $G_2$  does not yield itself to analysis (here we have only  $K$  independent fading variables, while in the uplink case we had  $K \cdot N$  fading variables, and when  $N$  is large we were able to model  $I_2$  as Gaussian). Therefore, we resort to simulations to estimate  $P_{out}$  as a function of cell loading and number of sensors, from which we can obtain the system capacity (maximum cell loading) as a function of cell loading and number of sensors. The results of these simulations are shown in Figs. 6–9 and are discussed in Section IV.

#### IV. SIMULATION AND NUMERICAL RESULTS

In all of our simulations and numerical results, we consider only the first two tiers of interfering cells, which means that  $K = 18$  cells. We assume that the voice activity factor  $v$  is 0.375. We assume that for adequate performance, the required BER is  $10^{-3}$  which corresponds to  $E_b/(N_o + I_o)$  of 7 dB. We also assume that the processing gain  $L$  is 128. Finally, we assume that  $\sigma_s$  is 8 dB.

For the uplink, the outage probability was computed using (22). The results are summarized in Fig. 3. From this figure, it is shown that by using antenna array to form narrow beams towards the desired mobiles, a many-fold increase in system capacity can be obtained. For example,

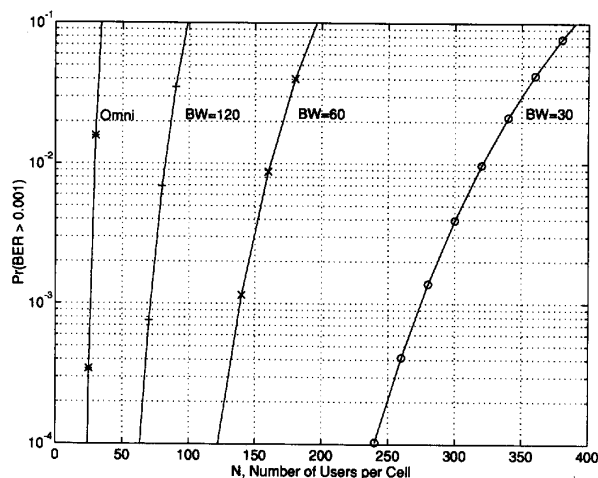


Fig. 3. Uplink outage probability as a function of beamwidth.

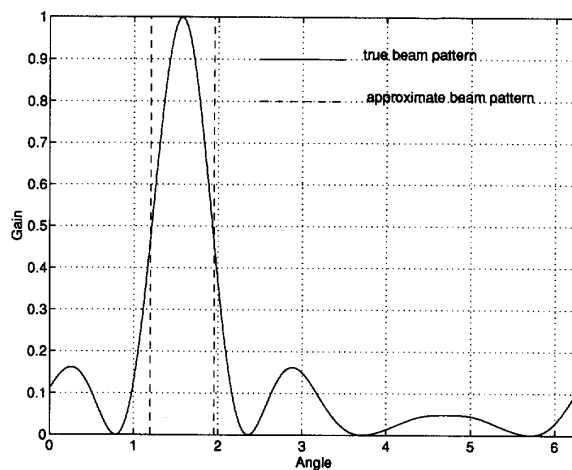


Fig. 4. Actual and approximate beam patterns.

for 0.01 outage probability, the uplink system capacity goes up from 31 users per cell for the single antenna case to about 320 users per cell if we use an array, such that we have beams with beamwidth of  $30^\circ$ . Also, to evaluate the accuracy of the approximation we used, we simulated the system (based on (14)–(16)) with a cell site circular antenna array with nine elements and radius equal to  $\lambda/2$  corresponding to half-power beamwidth of  $42^\circ$ . In fact, the beamwidth was taken to be slightly more than  $42^\circ$  to account for the interference energy picked up through the side lobes of the array pattern. Fig. 4 shows the actual array pattern versus the approximate pattern. In Fig. 5, we plot both the outage probability computed from (22) and from simulations, which indicate good agreement between the simulation results and the approximation.

For the downlink, results for the outage probability were obtained by simulations based on (31)–(33). A circular array with one, five, and seven elements and  $\lambda/2$  spacing was used in the simulation. For all other parameter values above, the histogram of  $E_b/(N_o + I_o)$  is obtained for each  $M$  and  $N$  value from 20,000 runs. In each run, 19 Rayleigh random variables with mean square value that have a log-normal distribution with  $\sigma_s = 8$  dB are generated, and the maximum of these is taken to be that of the desired mobile's cell site. Also, we assume that the mobile is positioned on the boundary between cells, which represents a worst case situation. Some of the generated histograms of  $G_1 + G_2$  are shown in Figs. 6, 7, and 8. The generated histograms are used to estimate the probability of outage as function of cell loading and number of sensors. These results are summarized in Fig. 9, which also shows a many fold increase in capacity by using antennas to form narrow beams towards the desired user. Note that as we mentioned before, if orthogonal codes are used on the downlink and in the case of no multipath, interference will be primarily due to outside cell interference and the corresponding cell loading  $N$  at which outage will occur will be larger.

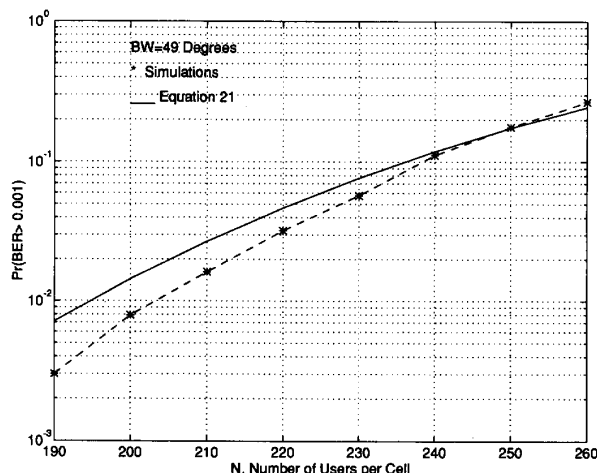
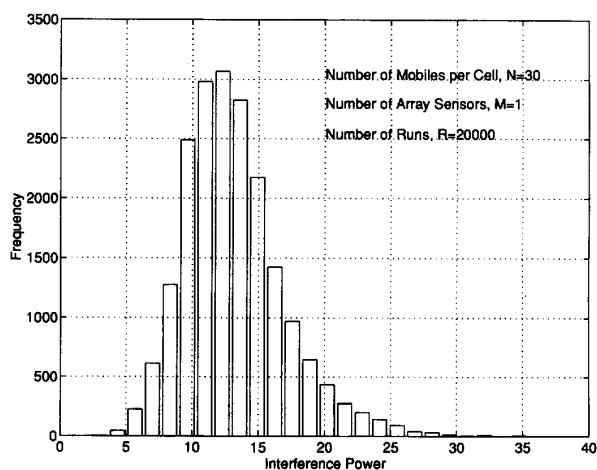


Fig. 5. Uplink outage probability: simulation vs. approximation.

Fig. 6. Histograms of  $G_1 + G_2$  for  $M = 1$ ,  $N = 30$ .

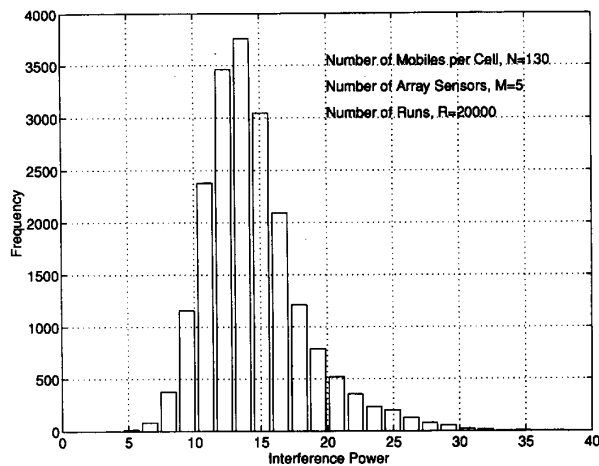
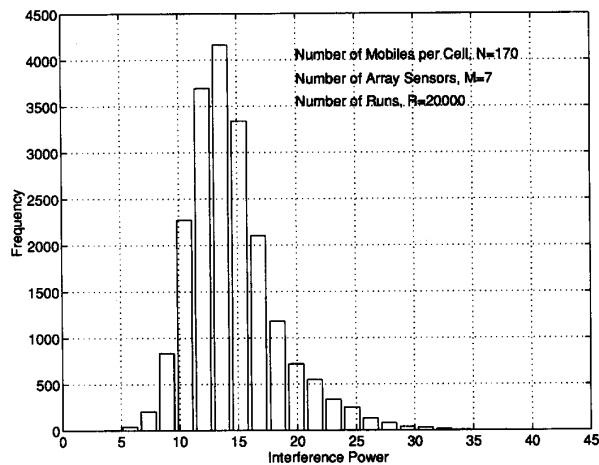
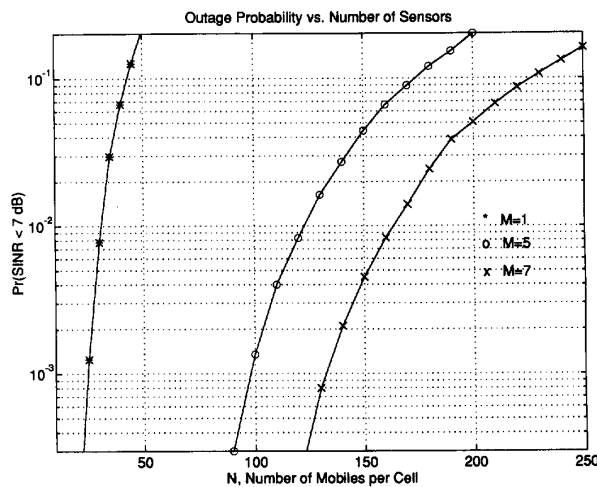
Fig. 7. Histogram of  $G_1 + G_2$  for  $M = 5$ ,  $N = 130$ .Fig. 8. Histogram of  $G_1 + G_2$  for  $M = 7$ ,  $N = 170$ .

Fig. 9. Downlink outage probability vs. number of array sensors.

## V. CONCLUSIONS

We have studied the capacity improvement for CDMA cellular communications systems with base-station antenna array for both uplink and downlink. The outage probability was evaluated as a function of cell loading, array parameters, fading and shadowing effects, and voice activity. Our analytical and simulation results show that there can be a substantial increase in system capacity by incorporating antenna arrays at the base-station. Our approach uses spatial processing to determine the dynamic spatial wavefront at the cell site and constructs a robust beamformer. Our model, used in this paper, does not include the effects of multipath which will be presented in a different paper.

## REFERENCES

- [1] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. Viterbi, L. A. Weaver, and C. Wheatly, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, no. 2, pp. 303-312, May 1991.
- [2] A. M. Viterbi and A. J. Viterbi, "Erlang capacity of a power controlled CDMA system," *IEEE J. Select. Areas Commun.*, vol. 11, no. 6, pp. 892-900, Aug. 1993.
- [3] B. Suard, A. Naguib, G. Xu, and A. Paulraj, "Performance analysis of CDMA mobile communication systems using antenna arrays," in *Proc. ICASSP'93*, vol. VI, Minneapolis, MN, pp. 153-156, April 1993.
- [4] A. F. Naguib, A. Paulraj, and T. Kailath, "Capacity improvement with base-station antenna arrays in cellular CDMA," in *Proc. 27th Asilomar Conf. on Signals, Systems and Computers*, vol. II, Pacific Grove, CA, pp. 1437-1441, Nov. 1993.
- [5] J. C. Liberti and T. S. Rappaport, "Reverse channel performance improvements in CDMA cellular communication systems employing adaptive antennas," in *Proc. GLOBECOM'93*, vol. I, pp. 42-47, 1993.
- [6] V. Weerackody, "Diversity for the direct-sequence spread spectrum system using multiple transmit antennas," in *Proc. ICC'93*, vol. III, Geneva, Switzerland, May 1993.
- [7] P. Balaban and J. Salz, "Optimum diversity combining and equalization in data transmission with application to cellular mobile radio—Part I: Theoretical considerations," *IEEE Trans. Commun.*, vol. 40, no. 5, pp. 885-894, May 1992.
- [8] S. C. Swales, M. A. Beach, D. J. Edwards, and J. P. McGeehn, "The performance enhancement of multibeam adaptive base station antennas for cellular land mobile radio systems," *IEEE Trans. Veh. Technol.*, vol. 39, no. 1, pp. 56-67, Feb. 1990.
- [9] J. Winters, J. Saltz, and R. Gitlin, "The capacity of wireless communication systems can be substantially increased by the use of antenna diversity," in *Proc. Conf. on Information Science and Systems*, vol. II, Princeton, NJ, pp. 853-858, Oct. 1992.
- [10] S. Anderson, M. Millnert, M. Viberg, and B. Wahlberg, "An adaptive array for mobile communication systems," *IEEE Trans. Veh. Technol.*, vol. 40, no. 1, pp. 230-236, Feb. 1991.
- [11] R. Kohno, H. Imai, and S. Pasupathy, "Combination of an adaptive antenna array and a canceller of interference for direct-sequence spread spectrum multiple-access system," *IEEE J. Select. Areas Commun.*, vol. 8, no. 4, pp. 675-682, May 1990.
- [12] R. C. French, "The effect of fading and shadowing on channel reuse in mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-28, no. 8, pp. 171-181, Aug. 1979.
- [13] R. O. Schmidt, "A signal subspace approach to multiple-emitter location and spectral estimation," Ph.D. dissertation, Stanford Univ., Stanford, CA 94305, 1981.
- [14] A. Paulraj, R. Roy, and T. Kailath, "Estimation of signal parameters by rotational invariance techniques (ESPRIT)," in *Proc. of 19th Asilomar Conf. on Circuits, Systems and Comp.*, 1985.
- [15] W.-P. Yung, "Direct sequence spread-spectrum code-division-multiple access cellular systems in Rayleigh fading and log-normal shadowing channel," in *Proc. ICC'91*, vol. II, pp. 871-876, 1991.
- [16] G. Xu, H. Liu, W. Vogel, H. Lin, S. Jeng, and G. Torrence, "Ex-

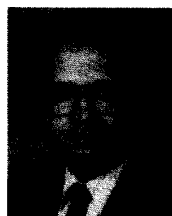
perimental studies of space-division-multiple-access schemes for spectral efficient wireless communications," submitted to SuperCom/ICC'94, May 1994.

- [17] J. H. Winters, "Signal acquisition and tracking with adaptive arrays in wireless systems," in *Proc. 43rd Veh. Technol. Conf.*, vol. 1, pp. 85-88, Nov. 1993.
- [18] D. Gerlach and A. Paulraj, "Base-station transmitting antenna arrays with mobile to base feedback," in *Proc. 27th Asilomar Conf. on Signals, Systems and Computers*, Pacific Grove, CA, pp. 1432-1436, Nov. 1993.
- [19] Y. Akaiwa, "Antenna selection diversity for framed digital signal transmission in mobile radio channel," in *Proc. VTC'89*, vol. 1, pp. 470-473, 1989.



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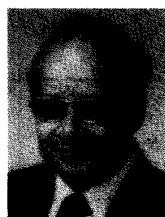


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