Graphs and Networks	Lecture 7

Percolation II

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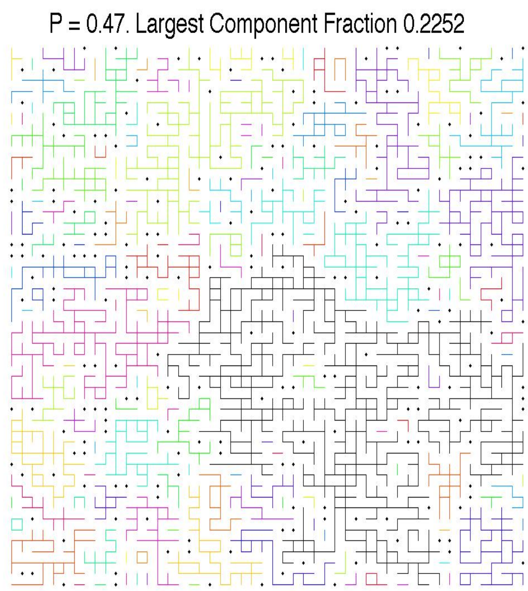
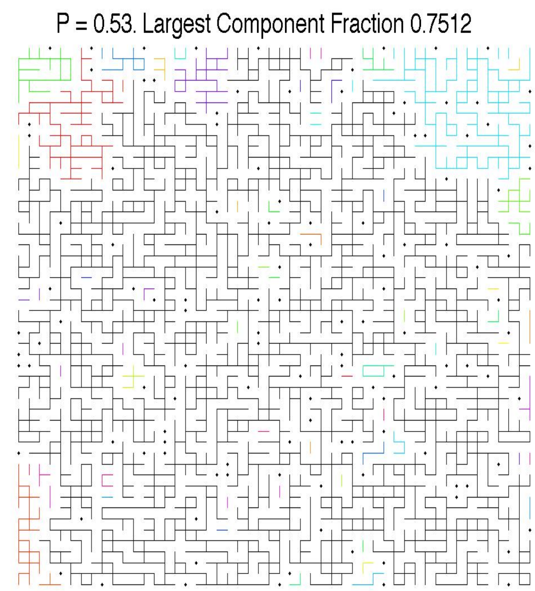
7.1 Disclaimer

These notes are not necessarily an accurate representation of what happened in class. They are a combination of what I intended to say with what I think I said. They have not been carefully edited.

7.2 Experiments

We began with an examination of percolation in the grid graph. You can see a demo of this by playing with the code `gridPercDemo`. We see that for $p < .5$, all components are small, and that for $p > .5$, there is a very large connected component.





To get an example without a clear threshold, I filled in all the cross edges in all the boxes at the bottom. This is realized in the code `gridPercDemo2`. This example really had two thresholds.

We then examines the behavior of the graphs that we studied in problem set 1. You can look at all of the plots of edge probability versus largest component by running the code `dataPercDemo`. We see that the only graphs that clearly exhibited threshold phenomena were the geometric graphs: the grid, Delaunay graphs, road networks, and (maybe) the k-nearest neighbor graphs.

7.3 Percolation in the Grid

I then proved two theorems about percolation in the grid: that for $p < 1/3$ there is no infinte component containing the origin, and that for $p > 2/3$, there is a constant chance of an infinite component containing the origin. You can find notes on this material in Lecture 16 of my notes from 2007.

7.4 Other things I mentioned

I also mentioned that percolation plays a role in the analysis of geometric graphs. In the first lecture, I drew graphs by choosing a subset of points in the plane, and then connecting all pairs of vertices whose distance from each other is less than some threshold. Graphs formed this way also exhibit a threshold phenomenon. If n points are chosen uniformly from $[0, 1]^2$, then there is some threshold radius of the form $r = c/\sqrt{n}$. For radii that are more than this, the graph probably has a large connected component. On the other hand, all components are probably small if the radius is smaller.

I also mentioned that one can analyze the giant component in Erdös-Rényi graphs in this way. If we choose an Erdös-Rényi graph with n vertices and $p = (1 + \epsilon)/n$, then it probably has one component that consists of a constant fraction of the vertices. On the other hand, if $p = (1 - \epsilon)/n$, then it is unlikely that any component has more than $c \log n$ vertices, for some constant c. The proof of this requires little more than the analysis of the Galton-Watson process that we covered last lecture. I wrote notes on it in Lecture 5 from 2010.

Finally, I mentioned that some people have also used percolation as a model for the adoption of behavior in networks. However, a paper of Goel, Watts and Goldstein [GWG12] suggests that such cascades are almost always shortlived.

References

[GWG12] Sharad Goel, Duncan J Watts, and Daniel G Goldstein. The structure of online diffusion networks. In *Proceedings of the 13th ACM Conference on Electronic Commerce*, pages 623–638. ACM, 2012.