Navigable Small World Graphs

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5.1   Overview

This lecture was based on Kleinberg's paper [Kle00b]. I can not improve upon it.

To learn more about what has happened since this breakthrough paper, I recommend [Kle00a], [Kle06], and [LNNK$^+$05]. For perspective on the Milgram experiments, I suggest [Gar79] and [Kle02].

5.2   Technical Point

The one useful mathematical fact that I wish to highlight from this lecture is the following: if you have an experiment that succeeds with probability p, then the *expected* number of independent trials that you need to repeat until you observe a success is $1/p$.

That is, the probability that you fail the first $i-1$ times and succeed on the ith is

$$p(1-p)^{i-1}.$$

So, the expected number of trials until the first success is

$$\sum_{i \geq 1} i \, \mathbf{Pr}\,[\text{the } i\text{th is the first success}] = \sum_{i \geq 1} p(1-p)^{i-1} = 1/p.$$

References

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[Kle06]   Jon Kleinberg. Complex networks and decentralized search algorithms. In *Proceedings oh the International Congress of Mathematicians: Madrid, August 22-30, 2006: invited lectures*, pages 1019–1044, 2006.

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