AMTH251a: Differential Equations and Orthogonal Decompositions

Fall 2005, Yale University

Lectures: MWF 10.30-11.20; BCT 508

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Text: Erwin Kreyszig, *Advanced Engineering Mathematics,* 8th Edition, John Wiley & Sons, 1999.
Supplementary material (to be distributed)

In this class, I will cover ordinary differential equations and systems (7 weeks), Fourier series and orthogonal functions (4 weeks), and applications of the Fourier eigenfunction method to boundary value problems and partial differential equations (2 weeks). Emphasis is on the mathematical techniques and concepts that are important in the engineering and physical sciences. By taking this course, a student will acquire a solid mathematical knowledge base for junior/senior classes in the applied sciences, as well as a proper intuition for later more theoretical courses, such as AMTH260, where rigorous proofs are constructed.

Prerequisites for the course are multivariate calculus and linear algebra; the latter could be taken concurrently. A natural continuation of the class is AMTH252b in the spring term which will include transforms, variational calculus, and more on partial differential equations.

Course Work and Grading Policy: There will be weekly homework assignments and a final exam. The final grade will be based 30% on the homework and 70% on the final exam.

Course Schedule (Tentative)

First-order differential equations

Week of 8/31	Differential equations: Basic concepts and ideas.
	Geometrical meaning of $y' = f(x, y)$. Direction fields and solution curves.
Week of 9/5	Linear 1st order equations. Applications.
	Separable equations. Differences between linear and non-linear equations.
	Existence and uniqueness theorem. Domain of validity of a solution.
Week of 9/12	Substitution methods and exact equations.
	More applications, e.g. population models, stability.
	Review of linear algebra and complex numbers.

Linear Differential Equations of Higher Order

Week of 9/19	Superposition principle for homogeneous equations. Concept of a basis.
	Existence-uniqueness theorem. Wronskians of solutions.
	Second-order homogeneous equations with constant coefficients.
	Modeling: Free oscillations of a mass-spring system.
Week of 9/26	Inhomogeneous equations and the method of undetermined coefficients.
	Modeling: Forced oscillations and resonance.
	Method of variation of parameters and Green's functions.

Systems of Differential Equations

Week of 10/3	Linear Systems with constant coefficients. Eigenvalues, eigenvectors.
	Stability and phase portraits.
Week of 10/10	Modeling: Normal modes in mechanics.
	Qualitative methods for non-linear systems. Examples in physics.

Fourier Series and Orthogonal Functions

Week of 10/17	Periodic functions. Trigonometric series. Fourier series and convergence.
Week of 10/24	Cosine and sine series. Half range expansions. More examples.
Week of 10/31	Finite and infinite-dimensional vector spaces.
	Orthogonal expansions in Hilbert spaces. Bessel's and Parseval's theorems.
Week of 11/7	Two-point boundary value problems. Sturm-Liouville Theory.

Application of Fourier Series to Partial Differential Equations

Week of 11/14	Vibrating strings and the 1D wave equation.
	Heat flow and conduction in a rod.
	Steady-state temperature and Laplace's equation.
Week of 11/28	Vibrating rectangular membrane and the use of double Fourier series.
	Introduction to other orthogonal functions. Review and wrap-up.

12/16/05 *Final Exam*